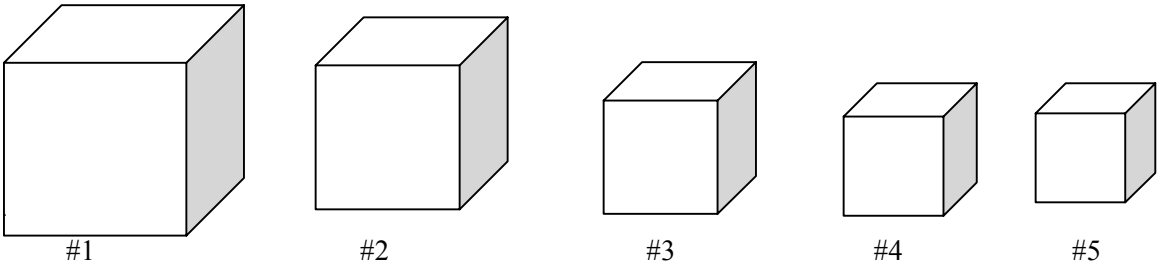
Direct Variation (Algebra I)

Goal: Understand direct variation, slope of a line and recognize and apply them to real-world problems.

(1) Exploration

Here are five blocks of the same type of rock. Sarah measured the dimensions and calculated the volumes of the blocks. She also weighed all the blocks. Unfortunately, Sarah forgot to write down the weight of block #5. However, Sarah believes that the weight of block #5 is 17 lbs based on her memory.



#1 #2 #3 #4 #5

Block	Volume (cubic feet)	Weight (pounds)	Weight/volume
1	2	96	48 lbs/cubic foot
2	1	48	48 lbs/cubic foot
3	$\frac{1}{2}$	24	48 lbs/cubic foot
4	$\frac{1}{3}$	16	48 lbs/cubic foot
5	$\frac{1}{4}$	12	48 lbs/cubic foot

1. Is 17 lbs a possible weight for block #5?

   At this point, students don't know this is a direct variation model but they can answer the question by the common sense: the bigger the block, the more it weighs. As the volume decrease the weight decreases also.

2. Use a scale to weigh block #5 and fill in the table.

   Check the answer for the question 1.

3. Use a ruler to measure the dimensions of each block and verify the volume for each block in the table.

   Review: Volume = Length × Width × Height

4. Complete the fourth column in the table and state your finding. Express the results as unit rates.

   Review: rate, unit rate, and unit analysis

$\frac{weight}{volume}$ = 48 lbs/cubic foot. This means that every cubic foot of the rock weighs 48 lbs.

Let us denote weight as y and volume as x, then $\frac{y}{x} = 48$

$$(\frac{y}{x}) \times x = 48 \times x$$

$$y = 48x$$

(2) Concept development

1. Direct variation

a. Definition: quantities y and x have direct variation if $y = b\,x$, where a non-zero constant b is called the constant of variation.

b. Do weight and volume have direct variation?

Give students time to discuss the question; give several examples and practice questions on how to identify constant variation question and find the constant of variation.

c. Using direct variation to find unknown.

Examples and practice questions

2. Graph the direct variation equation

Make a scatterplot of weight against volume and graph direct variation equation $y = 48\,x$ on the scatterplot. Do all of the points lie on the line?

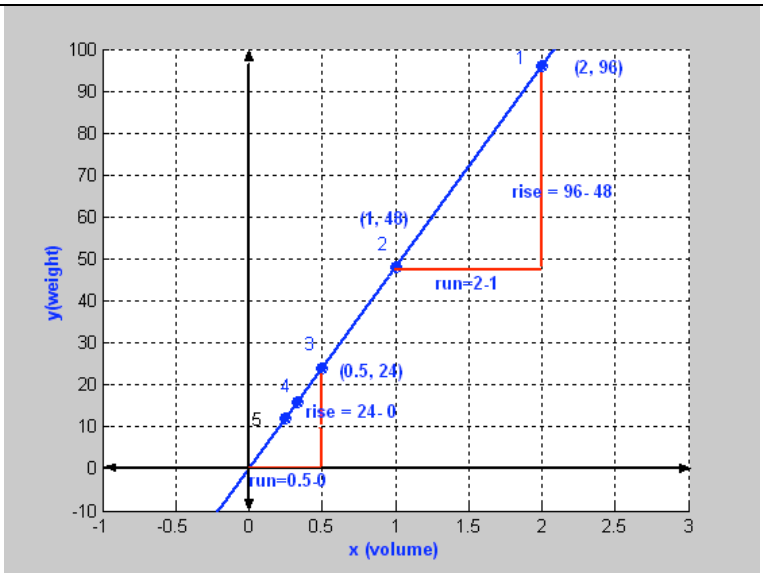
Review: coordinates, graphing scatterplots and linear equations.

3. The slope and y-intercept of a line.

a. Definition: The slope of a line is the ratio of vertical change (rise) to horizontal change (run) between any two points on the line. The slope measures the steepness of a line.

b. Definition: The y-intercept is the y coordinate where the line crosses the y-axis. For a direct variation equation $y = b\,x$, b is slope and intercept is 0 because the line always passes through the origin.

We focus on the equation $y = b\,x$. The equation $y = b\,x + a$ will be studied next time.



c. Verify the slope of the graph of $y = 48\,x$ is 48 by using a coordinate plane.

Use point 1 and point 2 to find the slope: $\text{Slope} = \frac{rise}{run} = \frac{96-48}{2-1} \frac{48}{1} = 48$.

Use point 3 and origin to find the slope: $\text{Slope} = \frac{rise}{run} = \frac{24-0}{0.5-0} = \frac{48}{1} = 48$.

These result also confirm that any two points on the line can be used to find the slope of a line. Several examples and practice questions on how to calculate the slope.

(3) Expansion

Give student several earth science based examples and practice questions.