What is the Mandlebrot Set?

The Mandlebrot set is the most complex, and most famous, object in mathematics. What you view on the screen when you start the computer program in the exercise is the Mandlebrot set.

The Mandelbrot set exists as an infinite swarm of points spread across a plain, represented in this instance by the pixels on the computer screen, only in the set there is the potential for an infinite number of pixels. The plain of points is itself the Acomplex number plain.@ It is defined by an infinite series of complex numbers.

A complex number is one with both a Areal@ and an Aimaginary@ part (e.g. $3\sqrt{2}$). The real numbers are spread along and define one side of the plain, and the imaginary numbers are spread along and define the second side of the plain. Thus, any point (pixel) on the plain is the result of a real and an imaginary portion, giving the complex number. The Mandlebrot set you see on the computer screen is thus in the complex plain.

But where do all the colors and geometric forms come from on the plain? They are generated by an iterated equation, that is an equation that is calculated, the results fed back into the equation which is then recalculated, over and over until something happens. Of course, this must be done for every point (pixel) on the plain (which is why it sometimes takes the computer time to generate the entire set.)

The procedure goes like this. Take a point on the complex number plain, place its value into the equation and iterate it 1000 times. If the number resulting from the equation settles down to one value, color the pixel black. If the number enlarges towards infinity then color it something else, say fast expanding numbers red, slightly slower ones magenta, very slow ones blue, and so on. Any color scheme is possible and one of the programs allows you to change the colors. Thus, if you have a sequence of pixels side by side, of different colors, that means that each of those values expanded toward infinity at a different rate in the iterated equation.

What is amazing about this are the complex geometric patterns that are created by calculating the Mandlebrot set. That is what you see on the computer screen. They consist of discs, swirls, bramble-like bushes, sprouts and tendrils spiraling away from a central disc. These are fractal geometric objects.

What is significant about this procedure for us is that the equations used are non-linear. That is, it is not possible to predict ahead of time how each complex number is going to behave in the iterated equation. You just have to calculate it and find out. That is, the Mandelbrot set is a chaos phenomena, and because of that the geometric figures that are generated are fractal in nature.

One more point, the complex plain of the Mandelbrot set has an infinite number of points, but a computer screen only has a finite number of pixels. Thus, the precision of each calculation is limited. However, when you enlarge a part of the set you are in essence increasing the number of pixels representing a smaller portion of the set. Since the complex plain is infinite this can be done indefinitely . . . as long as you have the computing power to carry it out, because the more you enlarge it the greater the number of decimal points you are dealing with. That is why in the programs you eventually cannot enlarge the set any more. You have reached the maximum computing resolution of the computer (about 15 decimal places.