

LABORATORY INSTRUCTION-RECORD PAGES

LABORATORY EXPERIMENTS WITH THE

X-next Equation

USING THE MODEL OF BRITISH BIOLOGIST ROBERT MAY, AND M.J. FEIGENBAUM
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PURPOSE

Of the several definitions for deterministic chaos, $X_{\text{next}} = rX(1-X)$ is the most informative and enlightening. Not the equation itself, of course, but its behavior. X_{next} is a system, meaning:

- L Its components are connected by positive and negative **feedback**.
- L It is **iterated**; calculated over and over
- L It is **recursive**; the output of each iteration is fed back into the next iteration. Meaning its present behavior is dependent on all its previous history.

The most important *thing to observe* studying this system is how its behavior changes as the local rules change. The most important *lesson to learn* is to recognize the state of each system just from its behavior: how stable it is, how sensitive dependent it is, how predictable its behavior is. Because the truth is, all open systems, of whatever nature, where ever they are found, behave exactly this way - including you.

Notes and Comments

- (When it comes time to do the formal experiments ***follow the instructions below***. They are designed to systematically lead you through a series of observations.
- (You may consult with your instructor about questions that come up in your experiments.
- (You may discuss your experiments with other class members, but the results in your "Record of Experimental Results" must follow logically from your own observations and specific experimental results.

OPENING THE X_{NEXT} PROGRAM

- ' X-next is available in the Geology Department computer lab, Miller 232. All the computers have the program. If you want a copy for your own computer, just ask.
- ' Turn the computer and monitor on. It will open to a Windows screen.
- ' When the Windows Program Screen comes open:
 - L Click on the **Evolutionary Systems** icon to bring that window up front.
 - L Double click the **X-next** icon to bring up the **X-next** program.

Both the Instructions for the experiments and the spaces for recording your experimental results are contained here.

Exploring the X-NEXT PROGRAM

- ‘ **X-next** is a Windows based program.
- ‘ If the program is unfamiliar to you observe the following:
 - L Values of X from 0.0 to 1.0 are along the vertical (Y) axis.
 - L Number of equation iterations are along the horizontal (X) axis. The actual number of iterations is in the lower left box on the screen.
 - L The “r” value and “initial x” value are also in the lower left.
 - L **Overwrite Previous** means that the curve for the last run will be erased before the new curve is drawn. Turning this to “No” allows you to compare curves.
 - L **Calculation Interval** - default is 5. Zero is the fastest speed.
 - L **View Output** gives you all the calculated values for the iterations you have.

Experiment One - X-next Attenuation

To attenuate is to diminish to extinction (zero) or to some stable value.

We observed during the lecture demonstration that up to a certain value of “r” the equation leads to attenuation; beyond that it might not. For 100 iterations find this value.

1. RUN ONE

- L Open X-next to the default values of **r=2.7** and **Iterations=100**.
- L OVERWRITE PREVIOUS = **No**
- L Increase values of “r” from **2.7** to **3.1** by values of **0.1**.

1. EXPERIMENTAL RECORD ONE – DESCRIBE WHAT HAPPENS TO THE ATTENUATION

For each value of r below sketch the behavior of the system for 100 iterations.

r = 2.7	r = 2.8	r = 2.9	r = 3.0	r = 3.1
Describe how the behavior of the system changes from r = 2.7 to 3.1				
Accept or Reject: based on the data in this experiment (iterations to only 100) r = 3.1 will never attenuate. Briefly describe the reasons for your acceptance or rejection.				

2. RUN TWO - ISOLATING ATTENUATION AT "r" = 3.0

- L** Set "r" at **3.0** and **iterations** at **500**.
- L** Set CALCULATION INTERVAL to zero (the fastest calculation speed).
- L** OVERWRITE PREVIOUS = **Yes**

2. EXPERIMENTAL RECORD TWO - ISOLATING ATTENUATION

Can you deduce from the graph for 500 iterations at "**r**" = **3.0** that attenuation is happening?

Go to VIEW OUTPUT. You will see the values of "r" for every iteration from 1-500. Go to iterations 480-500. To how many decimal places does the oscillation repeat. Write the last two values of X below only as far as they repeat decimal places exactly.

X at 499 =

X at 500 =

Conduct an experiment to discover the *fewest number of iterations* necessary to discover with confidence that $r=3.0$ attenuates to two decimal place; if it does attenuate write that iteration below.

Iteration of Attenuation =

Do you think $r=3.0$ ever completely attenuates? Conduct an experiment to discover the largest number of repeating decimal places you can reasonably find.

Iteration =

X =

3. RUN THREE - ISOLATING ATTENUATION AT "r" = 3.1

- L** Set "r" at **3.1** and **iterations** at **1000**.
- L** Set CALCULATION INTERVAL to zero (the fastest calculation speed).
- L** OVERWRITE PREVIOUS = **Yes**

3. EXPERIMENTAL RECORD THREE - ISOLATING ATTENUATION

Does the graph for 1000 iterations at "**r**" = **3.1** indicate that attenuation is happening, or does it appear that the system will continue to oscillate indefinitely between two values?

Go to VIEW OUTPUT. At 1000 iterations, to how many decimal places does the system oscillate between the two values; write the repeating oscillating numbers below?

X at 999 =

X at 1000 =

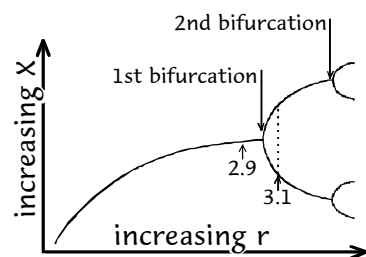
Find and record in this space the number of iterations necessary for " r " = 3.1 to stabilize.

Iterations to stabilize =

4. RUN FOUR - FINDING THE FIRST BIFURCATION

Clearly, " r "=2.90 leads to quick attenuation at 0.655, while r =3.1 leads to a period two oscillation at 0.558 and 0.764. The switch in behavior from one state to another state is called a **bifurcation**, and this switch from a period one to a period two is the first bifurcation.

We want you to find as closely as you can the exact value of " r " between 2.9000 and 3.1000 where it stops attenuating to one value and begins oscillation between two values (see drawing to right). Begin with values of " r " to two decimal places (e.g. 2.95), but increase the decimal places as you need. The two alternating values of X should repeat out at least to 4 decimal places.



- L Set ITERATIONS at 500.
- L Set CALCULATION INTERVAL to zero (the fastest calculation speed).
- L OVERWRITE PREVIOUS = Yes.

4. EXPERIMENTAL RECORD FOUR - FINDING THE FIRST BIFURCATION

Experiment with values between 2.9 and 3.1 (500 iterations only) trying to isolate where period 2 bifurcation initiates. Use as many decimal places for " r " as you need to do this; try to get X values repeating to 4 decimal places. The spaces below are to help you keep track of the X values for each run.

1. r = _____ X = _____, _____
 2. r = _____ X = _____, _____
 3. r = _____ X = _____, _____
 4. r = _____ X = _____, _____
 5. r = _____ X = _____, _____
 6. r = _____ X = _____, _____
 7. r = _____ X = _____, _____
 8. r = _____ X = _____, _____

9. r = _____ X = _____, _____
 10. r = _____ X = _____, _____
 11. r = _____ X = _____, _____
 12. r = _____ X = _____, _____
 13. r = _____ X = _____, _____
 14. r = _____ X = _____, _____
 15. r = _____ X = _____, _____
 16. r = _____ X = _____, _____

At what r value did the first bifurcation to period two oscillation take place, and how many iterations were required for attenuation to stop and the period two behavior become established.

$r =$

Iteration =

You were doing this to four decimal places. What would be required to get this value to more decimal places?

Experiment Two - X-next Second Bifurcation - Period Four Oscillations

5. RUN FIVE - FINDING THE PERIOD FOUR OSCILLATION

OK, to the best of our ability, in a reasonable amount of time, we have isolated the initiation of the first bifurcation. Now we want you to try to find the second bifurcation to a period 4 oscillation (see drawing on page 4).

Design this experiment any way you want to isolate this “ r ” value. We have provided below spaces to keep track of the values of X at the end of each experimental run. **Hint:** as you go to higher and higher bifurcation values it usually takes more and more decimal places of r .

5. EXPERIMENTAL RECORD FIVE – SECOND BIFURCATION; PERIOD FOUR OSCILLATION

1. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
2. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
3. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
4. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
5. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
6. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
7. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
8. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
9. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
10. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
11. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
12. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
13. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

14. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
15. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
16. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
17. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
18. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
19. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
20. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
21. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
22. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
23. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
24. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
25. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
26. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
27. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
28. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
29. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
30. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
31. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
32. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
33. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
34. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
35. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
36. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
37. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
38. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
39. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____
40. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

At what value of r does the period 4 oscillation begin. What are the values of the first period 4 out to as many decimal places as they repeated?

$r =$ _____ $X_1 =$ _____ $X_2 =$ _____ $X_3 =$ _____ $X_4 =$ _____

1. r = _____ X= _____; Iterations _____
2. r = _____ X= _____; Iterations _____
3. r = _____ X= _____; Iterations _____
4. r = _____ X= _____; Iterations _____
5. r = _____ X= _____; Iterations _____
6. r = _____ X= _____; Iterations _____
7. r = _____ X= _____; Iterations _____
8. r = _____ X= _____; Iterations _____
9. r = _____ X= _____; Iterations _____
10. r = _____ X= _____; Iterations _____
11. r = _____ X= _____; Iterations _____

12. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

13. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

14. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

15. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

16. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

17. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

18. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

19. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

20. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

21. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

22. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

23. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

24. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

25. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

26. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____, _____, _____, _____

27. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____ , _____ , _____ , _____

28. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____ , _____ , _____ , _____

29. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____ , _____ , _____ , _____

30. $r =$ _____ $X =$ _____, _____, _____, _____; Iterations _____

_____ , _____ , _____ , _____

At what value of r does the two-decimal-place, period 8 oscillation begin?

$r =$ _____

Now, for this value of r print out the last 40 values of “X” you were calculating (enough to fit on one sheet of paper). You can do this with the following steps.

➤ Go to VIEW OUTPUT in X-next.

➤ Select the last 40 values of “X” that come up in NotePad or WordPad (hold down the left mouse button and drag across to highlight what you want) and do an EDIT: COPY.

➤ Open (another) Notepad (START: PROGRAMS: ACCESSORIES: NOTEPAD) and paste these values into it.

➤ Go To: FILE: PRINT

On the printout, starting at the bottom, bracket the last 8 values of X. Now working up from the bottom bracket make sure the next eight values of X repeat the previous to at least two decimal places, and so on through the entire 40 values.

If all the numbers do not bracket into bundles of eight then go back and lower the r value a little and try it again. Keep going until you get bundles of eight to two decimal places.

Thought Question: If you wanted to get the bundles of eight to repeat to more decimal places than 2, should the r value be increased or decreased? Based on what you have learned so far make an educated prediction, and then run an experiment to test the prediction.

r should increase ? or decrease ? to get more decimal places.

Thought Question: Based on what you have observed so far of X_{next} what will happen to the behavior of the system as r creeps upward. Make an educated guess, and then do the following.

Do a series of calculations with slowly increasing values of r , examining each run to see what is happening to the bundles of 8. Examine the bundles for two things:

1. How the number of decimal places are changing with increasing r .
2. How the bundles of numbers are changing. As r creeps up do you begin to find any bundles of numbers that do not repeat the eight, say bundles of 2, 3, 4, or 5 numbers that are different from the eight? Keep going until you do and then bracket all the bundles of repeating eights, as well as the smaller bundles of numbers.

When it seems useful, print out the last series of calculations as before and examine the hard copy for the bundles of numbers. In the space below write the series of bundles in your last experimental run; e.g. 8, 8, 3, 8, 5, 8

Writing Assignment: Describe and explain how the behavior of X_{next} changes as r increases beyond the period eight oscillation.

Experiment Four - X-next Extinction, or Breaking the Bank

7. RUN SEVEN - EXTINCTION

This experiment asks you to explore the upper reaches of “r”. How precisely can you find the value of “r” that causes the x-next program to lock up? Use as many decimal places of r as necessary.

7. EXPERIMENTAL RECORD SEVEN - EXTINCTION

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

r = _____ Result _____

Value of r that results in extinction?

Can you provide an explanation for what is happening when X_{next} locks up?

In practical terms, can you think of any real world examples where, metaphorically, the “r” value got so high that the system crashed?

Experiment Six - X-next Sensitive Dependence on "R"

CREATING A DATA TABLE

For the next series of experiments you need to copy and compare the "View Output" of several experimental runs. We do this by pasting the results of each run into a spread sheet or table beside the "View Outputs" of other experimental runs. So, try this:

1. In X_{next} set $X = 0.4$ and $r = 4.0$ for **40 iterations** and calculate the outcome.
2. Open "View Output" and holding down the right mouse button, highlight all the data. Then do "Edit"Copy."
3. Do *one* of the following:
 1. Open Word and create a table with 5 columns. Paste the output you just copied into the left column. Set font size to 10 points.
 2. OR, open Excel and paste the output you have just copied into the left column, leaving one row at the top. Set font size to 10 points.
4. At the top of the column type in the r value you used to calculate this output.

Repeat procedure for each experimental run, pasting the output into the next column to the right.

ANALYZING YOUR DATA

Analyzing lists of numbers is tedious, but if we are going to understand the behavior of any system we must get into the data and figure out what they are saying. We are looking for two patterns in the data:

- < *Repetition - repeating values of X , not only for one value of r but also between different r 's.*
- < *Precision - the number of decimal places the X 's repeat to.*

Looking for Patterns:

Highlighting trends and patterns in the number can be done several ways. For example, the two number columns to the right have boxes drawn around bundles of numbers that exactly repeat to the same number of decimal places (large numbers indicate how many decimal places).

Numbers that cycle are bracketed together (e.g. $0.783, 0.583 \pm 0.783, 0.583 \pm 0.783 - 0.583$). These methods work well for low values of r .

For numbers generated at higher r values we are more interested in the **number of decimal places** the numbers repeat to.

For example, the columns to the left are for runs at 3.9000 and 3.9001, and we want to compare the second column with the first.

We did this simply by drawing a vertical zig-zag line through the second column separating decimal places to the left that repeat exactly with the first column from decimal places to the right that differ from the first column. This provides a quick, visual comparison of how similar the two systems are.

0.4000000	0.4000000	0.642580	0.784246
0.9360000	0.9360240	0.642998	0.541452
0.2336256	0.2335499	0.642743	0.794501
0.6982742	0.6981349	0.642947	0.522460
0.8216805	0.8219169	0.642784	0.798385
0.5714343	0.5708556	0.642915	0.515091
0.9550988	0.9554444	0.642810	0.799271
0.1672516	0.1660285	0.642894	0.513397
0.5431863	0.5400198	0.642827	0.799425
0.9677262	0.9687786	0.642880	0.513101
0.1218053	0.1179647	0.642888	0.799450
0.4171783	0.4058016		0.513053
0.9482482	0.9404181		0.799454
0.1913866	0.2185298		0.513045
0.6035555	0.6660378		0.799455
			0.513044
			0.799455
			0.513044
			0.799455

Sensitive Dependence - Creating the Data Table

8. RUN EIGHT - CREATING THE DATA TABLE

Follow Steps 1 through 4 in Creating a Data Table and construct a 5 column table as shown below for the following values of “r”:

< Set initial X = 0.4

< Run for 40 iterations

Note that 4.00001 repeats in the 2nd and 4th columns and 4.000001 repeats in the 3rd and 5th columns.

Compare 2 nd with 1 st and 3 rd with 1 st			Compare 5 th with 4 th	
1 st column ~ r = 4.000 00	2 nd column ~ r = 4.000 01	3 rd column ~ r = 4.000 001	4 th column ~ r = 4.000 01	5 th column ~ r = 4.000 001

9. RUN NINE - SENSITIVE DEPENDENCE - FIRST COMPARISONS

Following instructions “Analyzing Your Data” above compare both 4.00001 and 4.000001 with 4.00000 by drawing a vertical zig-zag line separating decimal places to the left that repeat exactly with the first column from decimal places to the right that differ from the first column.

9. EXPERIMENTAL RECORD NINE - SENSITIVE DEPENDENCE - FIRST COMPARISONS

L For how many iterations do these systems track out to 2 decimal places?

4.00000 with 4.00001

Iteration =

4.00000 with 4.000001

Iteration =

L For how many iterations do these three systems track out to 3 decimal places?

4.00000 with 4.00001

Iteration =

4.00000 with 4.000001

Iteration =

L At which iteration do these three systems begin to track together again?

Iteration =

10. RUN TEN - SENSITIVE DEPENDENCE - SECOND COMPARISONS

OK, 4.00001 and 4.000001 do not track well with 4.00000. But, perhaps 4.00001 and 4.000001 will track better. After all, 4.00001 is one in a hundred thousand, and 4.00001 is one in a million; surely these cannot differ that much.

10. EXPERIMENTAL RECORD TEN - SENSITIVE DEPENDENCE - SECOND COMPARISONS

L For how many iterations do these systems track out to 2 decimal places?

4.00001 with 4.000001

Iteration =

L For how many iterations do these three systems track out to 3 decimal places?

4.00001 with 4.000001

Iteration =

L At which iteration do these three systems begin to track together again?

Iteration =

11. RUN ELEVEN - EXPANDING THE OBSERVATION RECORD

Well, maybe 40 observations are not enough to discover the true nature of the system; maybe we just did not observe its behavior long enough. Maybe we need a better sample.

10. EXPERIMENTAL RECORD TEN - EXPANDING THE OBSERVATION RECORD

L Try the experiment for 4.00001 and 4.000001 out to **1000** iterations; print out only the last 40 iterations for each r value. Do these track any better than the first 40?

L Try the experiment for 4.00001 and 4.000001 out to **10,000** iterations; print out only the last 40 iterations for each r value and compare. Do these track any better?

12. RUN TWELVE - THE EYE IS QUICKER THAN THE MIND

Science prides itself on the accuracy and precision of numbers, of mathematics. And indeed ultimately we always work to find mathematical ways of expressing our concepts. But, this can be overdone. Our eyes, and the visual processing machinery in our brains, are in fact extraordinary instruments, and we should train them, and learn to trust them more. Our eyes in a fraction of a second can often tell us more than hours of number crunching.

12. EXPERIMENTAL RECORD TWELVE – THE EYE IS QUICKER THAN THE MIND

- L** Here we redo the experiment comparing $r = 4.00001$ and 4.000001 , calculated out to 40 iterations, only this time we are going to compare visually the results we got numerically in Experimental Run 10.
- L** Set $r = 4.00001$, $X = 0.4$, and iterations at 40. This time, however, after clicking “Go” hold down the ALT and PRINT SCREEN keys at the same time; this copies the image to the clipboard. Next open Word, WordPerfect, or some other program that can take the image and EDIT:PASTE the image onto a page.
- L** Now redo this procedure with $r = 4.000001$. (Note: if you reduce the images a little, click right button on mouse, you can get both on a $8\frac{1}{2} \times 11$ page.)
- L** Print out both pages and visually compare them. Observe that although the two runs track very close for the first dozen iterations or so, after that they quickly diverge. We now know that they will never closely track, that because of sensitive dependence these two systems have very different life histories. *For want of a nail . . .*
- L** Now pull out your data tables where you compared the tracking of $r = 4.00001$ and 4.000001 and compare them with these two spatial print outs. Can you see in both representations the same behavior just in different formats? because they are in fact different representations of the exact same systems.
Learning to recognize a system in different representations is a valuable skill.

You should be getting a sense by now that knowing how a system is behaving is crucial to understanding it. That low “r” systems are essentially dead. That moderate “r” systems, although they may fluctuate, will eventually dampen into something predictable; the only question is, how long? That high “r” systems are touchy and completely unpredictable.

That is, what is really important is being able to read a system, to put your finger on its pulse, to know its state of openness and from there predict its behavior. Without this skill you will never be able to understand why a system is behaving as it is.

Experiment Seven - X-next Sensitive Dependence on “X”

All the experiments until now have focused on r , the force that is driving the system. X on the other hand is comparable to “data.” Say, data you have gathered and are going to use as a basis for modeling the future behavior of a system. Or, data gathered by you at different times, or data gathered by different people in different parts of the system. The question is, “How much precision must data have?” Does a 5% error in data gathering matter? Or, 2%? Or, half a percent? And under what conditions?

13. RUN THIRTEEN - SAMPLING THE DATA

Create three data tables following the instructions in “Creating a Data Table.” Use the values of “ r ” and X below:

13. EXPERIMENTAL RECORD THIRTEEN - SAMPLING THE DATA

First set of tables:

L $r = 2.0$ for 10 iterations .

	1 st column	2 nd column	3 rd column
L “Gather” data at:	$X = 0.40$	$X = 0.42$	$X = 0.35$

Second set of tables:

L $r = 3.236$ for 15 iterations .

	1 st column	2 nd column	3 rd column
L “Gather” data at	$X = 0.40$	$X = 0.42$	$X = 0.35$

Third set of tables:

L $r = 4.0$ for 15 iterations .

	1 st column	2 nd column	3 rd column
L “Gather” data at	$X = 0.40$	$X = 0.40001$	$X = 0.42$

Examine each of the tables for the following:

L Compare 2nd column with 1st and compare 3rd with the 1st.

L Following the “Analyzing the Data” instructions above:

- < Compare number of decimal places: draw a vertical zig-zag line through the second column separating decimal places to the left that repeat exactly with the first column from decimal places to the right that differ from the first column. Compare the 3rd column with the first in the same way.
- < In each column bracket together numbers that cycle; e.g. period 2, period 4, etc. Note the number of decimal places the cycles repeat to.

14. RUN FOURTEEN - Analyzing the Impact of Errors in X

For each r , describe how the system's behavior changes with different X values for each of the following:

1. **Time to Stabilize:** How many generations (iterations) does it take each system to stabilize?
2. **Converge/Diverge:** whether the systems become more or less alike with time (iterations).
 - < Vertical line moves *left* systems become *more alike*; that is they converge.
 - < Vertical line moves *right* systems become *less alike*; that is they diverge.

14. EXPERIMENTAL RECORD FOURTEEN - ANALYZING THE IMPACT OF ERRORS IN X

$r = 2.0$	0.40	0.42	0.35
Time to Stabilize?			
Converge or Diverge?			
$r = 3.236$	0.40	0.42	0.35
Time to Stabilize?			
Converge or Diverge?			
$r = 4.0$	0.40	0.42	0.40001
Time to Stabilize?			
Converge or Diverge?			

Analysis One: Assume $X = 0.40$ is the "correct" data, the theoretical ideal (which, of course, you can never really know). A reading of $X = 0.42$ then is a 2% error, and $X = 0.35$ a 5% error. Do errors like this really make a difference in understanding or predicting the behavior of a system? Explain under what conditions and why?

Analysis Two: You and a team of other scientists are working on a dynamic system, gathering data and using that data to understand the state of the system at any point in time, or using that data to model the long term behavior of the system.

- A. Imagine that you gather data very carefully, and your average error is somewhere around half a percent, while a sloppy colleague gathers data with an average 5% error. Does it really matter? And under what conditions?

- B. Imagine that you are doing a longitudinal study of a system, that is studying it over an extended period of time under different conditions. Periodically you return and gather another data point, and then use these to compare the systems behavior through time. Does it matter whether the data is gathered with the same precision every time? Under what conditions?

We have spend some time developing X_{next} and exploring its behavior because the principles here apply to most systems. Sometimes it takes some hard thinking to recognize chaos principles in other systems, but once you begin to see them, they show up everywhere. Think about a system you are familiar with, or professionally working on.

- L Describe the system.
- L What is the nature of its data?
- L What is comparable to “r” in the system?
- L What is comparable to X in the system?
- L How does the system behave at low “r”?
- L How does the system behave at high “r”?
- L What conditions “tune” “r”?
- L How do you recognize them?
- L How much control do you have over them?

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