LABORATORY INSTRUCTION–RECORD PAGES

LABORATORY EXPERIMENTS WITH THE

The Logistic System

X-next Equation - Deterministic Chaos

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Fall, 2014 Version

PURPOSE

One of the central assumptions of classical science is that systems evolve to equilibrium. But, this is true only when driving energy is very low, and we are not curious enough to see what happens when systems are pushed. As energy dissipation increases, system behavior becomes more and more complicated and unpredictable. This behavior is termed ***deterministic chaos***. The purpose of this experiment is to experience this deterministic chaos first hand.

Deterministic chaos exists in many systems, but the logistic system— $X_{next} = rX(1-X)$ —is the most accessible. X_{next} is a logistic system, meaning:

☞ Its variables (rX and 1-X) are connected by positive and negative **feedbacks**. That is, the string of generated numbers (population sizes) are not random (in which case every event would be independent of the previous).

☞ It is **iterated**; calculated over and over for many generations..

☞ It is **recursive**; the output of each iteration is fed back into the next iteration. Meaning its present behavior is dependent on all its previous history.

The most important *thing to observe* studying this system is how its behavior changes as ***r***—the driving variable—changes.

Notes and Comments

☺ When it comes time to do the formal experiments ***follow the instructions below***. They are designed to systematically lead you through a series of observations.

☺ Consult with your instructor about questions that come up in your experiments.

☺ You may discuss your experiments with other class members, but the results in your "Record of Experimental Results" must follow logically from your own observations and specific experimental results.

OPENING THE X_{NEXT} PROGRAM

❑ X-next is available in the Geology Department computer labs as an Excel spread sheet. All the computers have the program. If you want a copy for your own computer, just download it to a memory stick, or e-mail it to yourself.

❑ Turn the computer and monitor on. It will open to a Windows screen.

❑ When the Windows Program Screen comes open:

  ☞ Go to: My Computer\T\AlifeProg\Xnext\

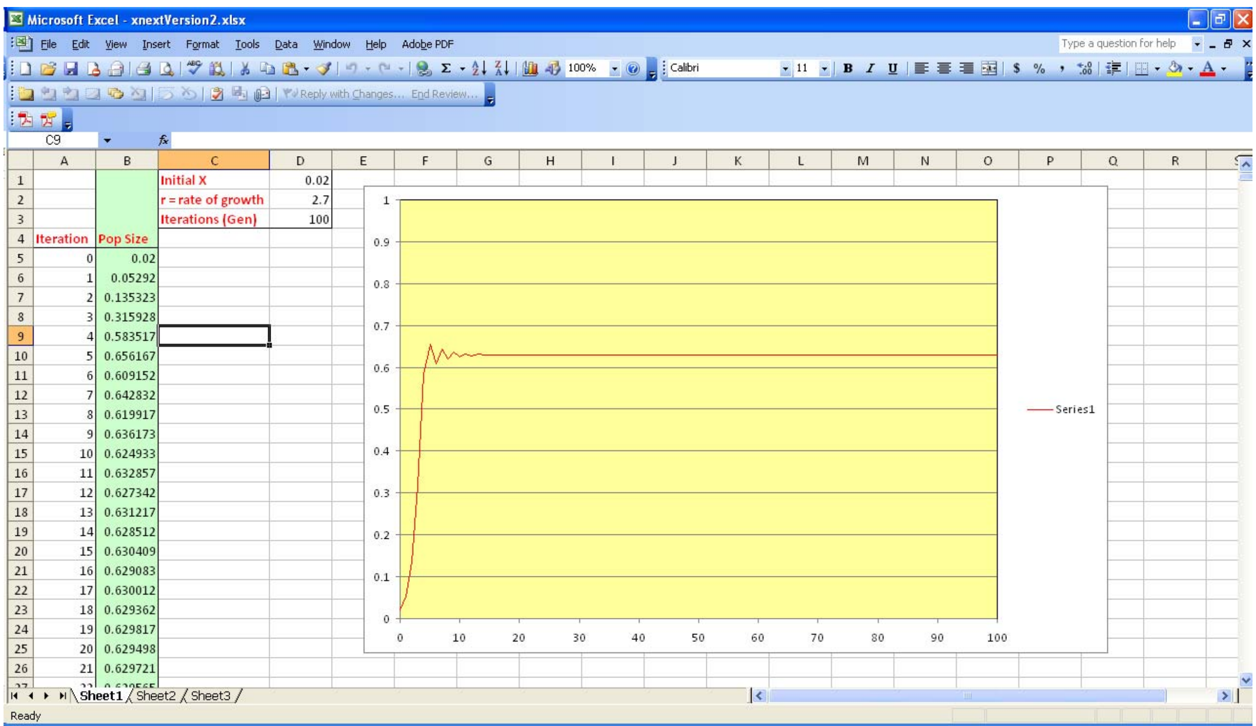
  ☞ Double click the **xnextVersion2.xlsx** shortcut to open the program;.

❑ **Copy the xnextVersion2.xlsx file onto the desktop and work with it there (e-mail copy to yourself).**

Both the Experimental Instructions data sheets for recording your experimental results are below.

Exploring the X-next Program

- ❑ **X-next** is a spread sheet based program in Excel (screen capture below).
- ❑ If the program is unfamiliar to you observe the following:
  - ☞ The box in yellow is the time series graph and plots out the changing population size generation by generation.
  - ☞ Values of X—population size—from 0.0 to 1.0 are along the vertical (Y) axis; do not change this.
  - ☞ Number of equation iterations—or generations—are along the horizontal (X) axis. You can change the X axis number of generations by right clicking on it and doing "Format Axis." Default is 100 generations.
  - ☞ The "initial X", "r" value" and number of generations are in the D column. You change "r" and number of generations for various experiments.
  - ☞ ***Column A*** is the number of iterations; that is, generations that will be plotted.
  - ☞ ***Column B*** is the population size (value of X) for each generation. There are times when we ask you to check actual population sizes.



Experiment One - X-next
Attenuation

To attenuate is to diminish to extinction (zero) or to some stable value.

We observed during the lecture demonstration that up to a certain value of "r" the equation leads to attenuation; beyond that it might not. **For 100 iterations find this attenuation value (X) for the different *r* values**.

1. Run One

- ☞ Open X-next to the default values of **r=2.7** and **Iterations=100**.
- ☞ Increase values of **"r"** from **2.7** to **3.1** by values of **0.1**; observe and record results.

1. Experimental Record One – Describe What Happens to the Attenuation

<table>
<tr><td colspan="5">For each value of r below sketch the behavior of the system for 100 iterations.</td></tr>
<tr><td>r = 2.7</td><td>r = 2.8</td><td>r = 2.9</td><td>r = 3.0</td><td>r = 3.1</td></tr>
<tr><td colspan="5">Describe how the behavior of the system changes from r = 2.7 to 3.1</td></tr>
<tr><td colspan="5">Accept or Reject: based on the data in this experiment (iterations to only 100) r = 3.1 will never attenuate. Briefly describe the reasons for your acceptance or rejection.</td></tr>
</table>

Run Two - Isolating Attenuation at "r" = 3.0

- ☞ Set **"r"** to **3.0** and **iterations** to **500** (right click horizontal scale).
- ☞ Right click the X axis to change the generation number.
- ☞ If the program does not plot out all 500 generations, right click the graph line, click "Select Data", and you will see this: =Sheet1!$A$5:$B$105. Change the last 105 to 505 and it will plot 500 generations.

2. Experimental Record Two – Isolating Attenuation

Can you deduce from the graph for 500 iterations at **"r" = 3.0** that attenuation is happening?

Go to the green column of population sizes. You will see the values of “X” for every iteration. Go to iterations 480-500. To how many decimal places does the oscillation repeat. Write the last two values of X below only as far as they repeat decimal places exactly.

X at 499 = X at 500 =

Conduct an experiment to discover the *fewest number of iterations* necessary to discover with confidence that r=3.0 attenuates to two decimal place; if it does attenuate write that iteration below.

Iteration (generation) of Attenuation =

Do you think r=3.0 ever completely attenuates? Conduct an experiment to discover the largest number of repeating decimal places you can reasonably find.

Iteration (Generation) = X=

5. Run Three - Isolating Attenuation at “r” = 3.1

- ☞ Set **”r”** at **3.1** and **iterations** at **1000**.
- ☞ Right click the X axis to change the generation number to 1000.
- ☞ IF the program does not plot out all 1000 generations, right click the graph line, click “Source Data”, and you will see this: =Sheet1!$A$5:$B$105. Change the last 105 to 1005 and it will plot 1000 generations.

3. Experimental Record Three – Isolating Attenuation

Does the graph for 1000 iterations at **“r” = 3.1** indicate that attenuation is happening, or does it appear that the system will continue to oscillate indefinitely between two values?

Go to COLUMN B; X VALUES. At 1000 iterations, to how many decimal places does the system oscillate between the two values; write the repeating oscillating numbers below?

X at 999 = X at 1000 =

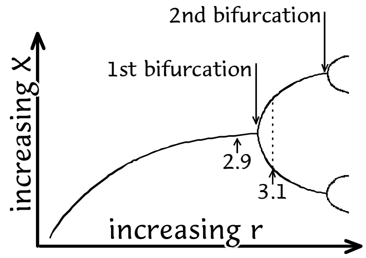
Find and record in this space the number of iterations necessary for **“r” = 3.1** to stabilize.

Iterations (generations) to stabilize =

4. Run Four - Finding the First Bifurcation

Clearly, "r"=2.90 leads to quick attenuation at 0.655, while r=3.1 leads to a period two oscillation at 0.558 and 0.764. The switch in behavior from one state to another state is called a ***bifurcation***, and this switch from a period one to a period two is the first bifurcation.

We want you to find as closely as you can the exact value of "r" between 2.9000 and 3.1000 where it stops attenuating to one value and begins oscillation between two values (see drawing to right). Begin with values of "r" to two decimal places (e.g. 2.95), but increase the decimal places as you need. The two alternating values of X should repeat out at least to 4 decimal places.



☞ Set ITERATIONS at 500.

☞ **CONFUSED? Try this:** Think about it this way; the system's behavior changes as the r value goes up. At one r value it attenuates to 1 population size. At another r value it oscillates between 2 population sizes. We want to find out at what r value it switches from attenuation to 1 size to oscillation between 2 sizes. The way you do this is to either slowly increase r until you see the switch from 1 to 2, or you slowly decrease r until you see the switch from 2 to 1.

The difficulty is, this transition is not exact. In fact it can be virtually impossible to find exactly. So, you will only be able to get it approximately. And, we are not looking for a "right" r value, especially when you get to the next Experiment 2 where the transition is from oscillation 2 to oscillation 4. The other thing is, as you hone in closer and closer to the transition it takes more and more decimal places of r to find the transition. So you may start off with something like 2.9 and end up with something like 2.9575 (not the real numbers).

4. Experimental Record Four – Finding the First Bifurcation

Experiment with values between 2.9 and 3.1 (500 iterations only) trying to isolate where period 2 bifurcation initiates. Use as many decimal places for "r" as you need to do this; **try to get X values repeating to 4 decimal places**. The spaces below are to help you keep track of the X values for each run.

1.	r = ________	X = __________, __________	9.	r = ________	X = __________, __________
2.	r = ________	X = __________, __________	10.	r = ________	X = __________, __________
3.	r = ________	X = __________, __________	11.	r = ________	X = __________, __________
4.	r = ________	X = __________, __________	12.	r = ________	X = __________, __________
5.	r = ________	X = __________, __________	13.	r = ________	X = __________, __________
6.	r = ________	X = __________, __________	14.	r = ________	X = __________, __________
7.	r = ________	X = __________, __________	15.	r = ________	X = __________, __________
8.	r = ________	X = __________, __________	16.	r = ________	X = __________, __________

At what r value did the first bifurcation to period two oscillation take place, and how many iterations were required for attenuation to stop and the period two behavior become established.

r = Iteration (generation) =

You were doing this to four decimal places. What would be required to get this value to more decimal places?

Experiment Two - X-next

Second Bifurcation - Period Four Oscillations

5. Run Five - Finding the Period Four Oscillation

OK, to the best of our ability, in a reasonable amount of time, we have isolated the initiation of the first bifurcation. Now we want you to try to find the second bifurcation to a period 4 oscillation (see drawing on page 5).

Design this experiment any way you want to isolate this "r" value. We have provided below spaces to keep track of the values of X at the end of each experimental run.

This is like the last experiment you just finished. You have to either increase r values from the bottom and/or decrease r values from the top until you narrow in on the r values where the bifurcation switch takes place.

Hint: as you go to higher and higher bifurcation values it usually takes more and more decimal places of r.

5. Experimental Record Five – Second Bifurcation; Period Four Oscillation

1. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
2. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
3. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
4. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
5. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
6. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
7. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
8. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
9. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
10. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
11. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
12. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______
13. r = _______ X = ___________, ___________, ___________, ___________; Iterations _______

(Continue table next page)

14. r = _______ X = _____________, ____________, _____________, ____________; Iterations _______
15. r = _______ X = _____________, ____________, _____________, ____________; Iterations _______
16. r = _______ X = _____________, ____________, _____________, ____________; Iterations _______
17. r = _______ X = _____________, ____________, _____________, ____________; Iterations _______
18. r = _______ X = _____________, ____________, _____________, ____________; Iterations _______
19. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
20. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
21. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
22. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
23. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
24. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
25. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
26. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
27. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
28. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
29. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
30. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
31. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
32. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
33. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
34. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
35. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
36. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
37. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
38. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
39. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______
40. r = ________ X = _____________, ____________, _____________, ____________; Iterations _______

At what value of r does the period 4 oscillation begin. What are the values of the first period 4 out to as many decimal places as they repeated?

$r =$ $X_1 =$ $X_2 =$ $X_3 =$ $X_4 =$

Experiment Three - X-next

Extinction, or Breaking the Bank

6. Run Six - Extinction

This experiment asks you to explore the upper reaches of "r". **How precisely can you find the value of "r" that causes the x-next program to crash to extinction?** Use as many decimal places of r as necessary.

☞ Note that in Column B, the first time a negative number shows up the program has crashed. Population cannot drop below 0.0, extinction.

☞ Further Description: at an r of 4.0 the system will behave wildly, but do so for as many generations as you run it. For r of 4.1 the system self-destructs - after a few generations it just stops running because one of the population sizes drops below zero - or extinction. Question is, where between 4.0 and 4.1 does this breakdown take place. As with experiment 3, it takes more and more decimal places to find it.

6. Experimental Record Six – Extinction

r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________
r = _______ Result ______________________	r = _______ Result ______________________

Value of r that results in extinction?

Can you provide an explanation for what is happening when X_{next} locks up?

In practical terms, can you think of any real world examples where, metaphorically, the "r" value got so high that the system crashed?

Experiment Four - X-next
Sensitive Dependence on "r"

7. Run Seven - "For want of a nail . . . "

Science prides itself on the accuracy and precision of numbers, of mathematics. And indeed ultimately we always work to find mathematical ways of expressing our concepts. But, think about other science courses you have taken. To how many decimal places do you usually do calculations? Two? Three? In classical science this is usually enough. In Chaos and Complex Systems theories this is not nearly good enough, as this experiment demonstrates.

We could make this comparison by a lot of number crunching, but in this case our eyes, and the visual processing machinery in our brains, are by far the most powerful tool. In a fraction of a second our eyes can tell us more than hours of number crunching.

7. Experimental Record Seven – For want of a nail . . .

- ☞ Here we compare the behavior of the logistic system at two values that differ by only a millionth: r = 4.000001 and 4.000002, calculated out to 40 iterations.
- ☞ Set r = 4.000001, X 0.2, and iterations at 40. After clicking "Go" copy the graph and paste it into Word or some other program that can take the image.
- ☞ Now redo this procedure with r = 4.000002.

- ☞ Print out these page(s) and visually compare them, OR.
- ☞ If possible, lay the second page at r = 4.000002 on top of the first so the time series graphs line up exactly, and hold them up to a window. Trace in one color or in one way only the parts of the graph that matches up exactly with the one under it. Then trace in another color, or in another way, where the 4.000002 diverges from the path of the 4.000001 run.

- ☞ Observe that although the two runs track very close for the first dozen iterations or so, after that they quickly diverge. We now know that they will never closely track, that because of sensitive dependence these two system have very different life histories. *For want of a nail . . .*

☺ F ☺ i ☺ n ☺ i ☺ s ☺