

LABORATORY INSTRUCTION-RECORD PAGES

Laboratory Experiments With The X-next The Logistic Model and **Bifurcation Diagrams**

Using the Model of British Biologist Robert May, and M.J. Feigenbaum
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ENVT 200 - Environmental Systems Theory
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Purpose

We have explored the X_{next} (logistic) system, but the X_{next} program you used only allows us to examine one r value at a time, through some number of iterations or generations. And we know from the last experiment that if you want to compare different behaviors at different r values it is cumbersome. You have to line up and compare a lot of time series diagrams. And as the r gets up to 3.5 and above the behavior becomes very complicated making comparisons difficult. There is an easier way to compare changes in behavior from $r = 1.0$ to $r = 4.0$ all at once. This is called a *bifurcation diagram* which we need to calculate next.

Notes and Comments

- ☺ When it comes time to do the formal experiments *follow the instructions* below. Do what they tell you to do in exactly the way they tell you to do it and no problems will arise.
- ☺ You may consult with your instructor about questions that come up in your experiments.
- ☺ You may discuss your experiments with other class members, but the results in your “Record of Experimental Results” must follow logically from your own observations and specific experimental results.

Opening the Bifurcation and Grapher Programs

These experiments require two programs, **Xnextbif.exe** to calculate the data, and a graphing program to plot the data (we use **Grapher**, but other graphing programs work). **Xnextbif.exe** has an icon under the Alife folder. The **Grapher** shortcut moves; we will have to tell you where to find it. Once you know how to run the programs they are easy, but getting started may be a hassle. We will lead you through the first experiment to reduce the hassles. Below, however, are two boxes, one for the procedures with **Xnextbif.exe** and the other for **Grapher**.

- Xnextbif.exe** is available in the Geology Department computer lab. On the desktop click Alifeprog, then go to folder Xnextbif. Do a copy this folder; we want to past it someplace else.
- Grapher** is a commercial program, and unless you purchase a copy, is available only in Geology computer lab.

Both the Instructions for the experiments and the spaces for recording your experimental results are contained here.

Creating the Data File With the X-next Bifurcation Program

Running the XnextBif.exe Program

- ❑ **X-nextbif** runs in a DOS window (Disc Operating System; an extremely old, first generation Windows control program); screen capture below. We suggest you create a folder for the experiment and copy and paste X-nextbif into that folder. We will talk you through this.

NOTE: DOS is very persnickety (a.k.a. anal). Everything has to be just exactly right, or it gives you a dumb look. It will not accept any file or directory names with punctuation or blank spaces. DOS file names are also limited to 8 characters. And unlike windows you cannot return to a previous line if you make a mistake. You have to start over, or backspace everything.

- ❑ On either a memory stick, or the S:\envt drive, (1) create a folder and give it your name—first name and first letter of last name (no more than 8 characters). (2) Copy and paste the X-next bif program into the folder; we will tell you where to find it. (3) All your data files will be stored in this folder.

Very Important:

When creating folder/file names, keep # of characters to < 8. Use NO spaces.

- ❑ Generate a data file. You are prompted to enter the parameters of the calculation.

- ☞ Rmin/Rmax - range of r values in the run; **1.0 to 4.0**; r values > 4.0 cause the system to crash

- ☞ Increment for r - use **.001** as a default in thousands of an r ; this provides enough detail to see the fine structure

- ☞ Initial value of X - use default of **0.2** unless directed otherwise

- ☞ X 's per run - use **2000**; this is enough to give most values of r time to attenuate about as much as they are going to attenuate.

- ☞ X 's to plot - **25**; this is the memory hog in the program; we say here how many points to plot for each 0.001 increment of r . If we plot the last 200 iterations, at low values of r these all plot on top of each other because the system quickly attenuates to a single value and just keeps repeating it, plotting the same point over and over. At higher r values, where attenuation does not occur the 200 points (values of X) scatter vertically. The key is to find a value of X 's per run high enough to show the behavior of the system, but not so high to make the files too large to be managed. Plotting 100 X 's gives a good ghost image of the bifurcation diagram; 200 shows some of the details.

- ☞ Name of output file - name the file with your first name and first letter of last name, like lynnf.dat. If you have the X-nextbif program in the same folder you plan to store your data files this is all you need. Hit return and program will begin calculating. See experiment 1 for more detailed instructions.

```

MS-DOS XnextBif
Auto
Calculate X-Next Bifurcation Diagram
as fctn of Rmin & Rmax --wf, 3iv01 ver 1.4

Enter Rmin Rmax: 2.5 4.0
Enter Increment for R: .001
Enter initial X value: .2
Enter # of X's per run: 5000
Enter # of X's to plot (last N of the run): 500

Name of the OUTPUT file : bif6.dat_
  
```

Be sure to add the .dat extension.

Once Xnextbif.exe is done it prints the output to the folder.

DO NOT Print to Screen

Plotting the Data in A Graphing Program

Creating and Printing a Bifurcation Diagram with Grapher

Grapher is a commercial Windows based program, and is available on all the computers in the lab, but you can use any program that allows you to build diagrams from raw data, such as Excel or Quattro Pro.

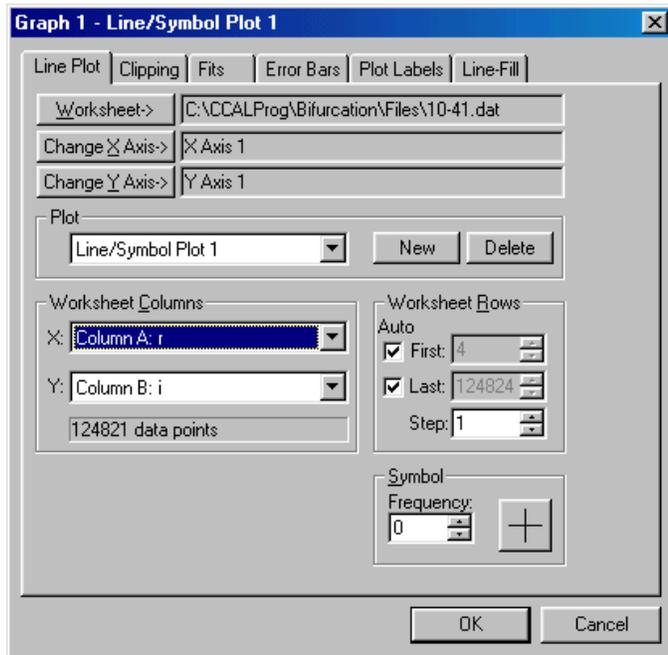
As with most programs that have a lot of power and options Grapher is complex to use, but once you know what you want to do, the protocol is not that complex.

Find the Grapher icon in the AlifeProg folder on the desktop and double click it to open the program. It should open with a new page. Go to **VIEW:PAGE** to enlarge and center it if it is not already.

Load the data file: go to **GRAPH: NEW GRAPH: LINE/SYMBOL** and under "Look in" find your data file and double click it. Grapher will load the data and open the window to the right.

In the GRAPH 1 - LINE/SYMBOL PLOT 1 window:

- ▶ Go to **LINE-FILL: LINE PROPERTIES** and set the Style to **INVISIBLE** (first choice below the first line).
- ▶ Back to **LINE-PLOT**: go to **SYMBOL** (lower right), click on cross hair, **select the black dot** and set the **size to .01 inch**. Also set **frequency to 1**.
- ▶ Click OK and Grapher graphs your data into a bifurcation diagram.
- ▶ If you want to change anything about the graph, select it by clicking on it and then right click to **PROPERTIES**.
- ▶ **LABEL YOUR GRAPH**. In the left side tool bar click on the letter A, then click it over the page and a text window opens. Type the following:
 1. Your name
 2. Increments =
 3. Initial X =
 4. X's per run =
 5. X's to plot =
- ▶ Click OK and that text shows up on the diagram. You may move it anywhere you want.
- ▶ **OR ALTERNATIVELY** - Use **WINDOWS EXPLORER** to find the data file in its directory. Double click the file and it will open in NotePad. **COPY** the experimental parameters at the top and paste them into Grapher.



Printing your File in 2 Steps

From Grapher first print your graph **as a pdf file** to your folder. Then print the pdf file to the lab printer - it goes much faster but may still take a couple of minutes.

Experiment One - Bifurcation Diagrams

The Basic Bifurcation Diagram

- 1. Run One - Complete Bifurcation Diagram** (in lab we will talk you through this first example).
- ☞ Open XnextBif and fill in the following parameters
 - ▶ Enter Rmin Rmax **1.0 4.0**
 - ▶ Enter Increment of r **.001**
 - ▶ Enter Initial X value **.2**
 - ▶ Enter # of X's per run **2000**
 - ▶ Enter # of X's to run (last N of run): **25**
 - ▶ Name and path of Output File: We talk you through this, but if you have copied the X-next bif program into your folder you only need give the run a simple name, like exp1.dat ¹
 - ▶ Output to Screen **No**
 - ▶ Strike return key to run the program.
 - ☞ Load data file into Grapher following the instructions above.
 - ☞ **PRINT file as a pdf to your folder, then print the pdf to the printer.**

1. EXPERIMENTAL RECORD ONE – COMPLETE BIFURCATION DIAGRAM

Explain in writing exactly what this diagram represents (in terms your younger sibling would understand; if you don't have a younger sibling, borrow one)

Label the 1st, 2nd, and 3rd bifurcations on the diagram. Can you discriminate any higher bifurcations?

Do bifurcations come *faster* or *slower* with increasing r ? Can you figure out a rate at which the rate of bifurcations change?.

¹ The X-nextbif program is designed to put the file in the same folder the program is in. If you plan to put the data files in another folder you must specify the entire path, like S:\ENVT-DataFiles\your folder name\file name.dat .

Remember the Three Phases of Science? Bracket and label on the bifurcation diagram the zone you think represents **Simple systems** (that which settles down to equilibrium).

Bracket and label on the bifurcation diagram the zone you think represents Phase Three of the Three Phases of Science, that is the realm of *organized complexity* (also called *deterministic chaos*).

Study the bifurcation diagram. Identify at least three interesting or unusual things about, or in, the diagram that indicate interesting and/or unexpected behavior of the system.

- 1.
- 2.
- 3.

Draw boxes around two parts of the diagram where you would like to see more detail.

Experiment Two - Bifurcation Diagrams Honing in on the Complexity Realm

2. Run Two - Honing in on the Complexity Realm

☞ Create and print a bifurcation diagram with the following parameters

- ▶ Enter Rmin Rmax **3.45 3.7**
- ▶ Enter Increment of r **.001**
- ▶ Enter Initial X value **.2**
- ▶ Enter # of X's per run **2000**
- ▶ Enter # of X's to run (last N of run): **200**
- ▶ Name and path of Output File: **same as before**
- ▶ Output to Screen **No**

☞ Load data file into Grapher following the instructions above.

☞ **PRINT file as a pdf to your folder, then print the pdf to the printer.**

2. EXPERIMENTAL RECORD TWO – HONING IN ON THE COMPLEXITY REALM

Return to the Diagram for Experiment One and draw two vertical lines through it showing the range of r this second diagram represents.

Describe any new features that appear in this more detailed diagram that are not clearly visible in the first diagram.

Experiment Three - Bifurcation Diagrams

Bifurcation Windows in the Chaos Realm

The last diagram narrowed the r range down to 3.45 to 3.7. Let's narrow it down even more, to 3.6 -3.65.

3. Run Three - Bifurcation Windows in the Chaos Realm

☞ Create and print a bifurcation diagram with the following parameters

- ▶ Enter Rmin Rmax **3.60 3.65**
- ▶ Enter Increment of r **.0001**
- ▶ Enter Initial X value **.2**
- ▶ Enter # of X's per run **2000**
- ▶ Enter # of X's to run (last N of run): **100**
- ▶ Name and path of Output File **Same as last**
- ▶ Output to Screen **No**

☞ Load data file into Grapher following the instructions above.

☞ FILE: PRINT

3. EXPERIMENTAL RECORD THREE – BIFURCATION WINDOWS IN THE CHAOS REALM

On the diagram for the Second experiment draw two vertical lines indicating the 3.6 to 3.65 range, just so you have an appreciation of the new scale of observation we are at.

Run Three - what new feature appears at this higher magnification that are not (so easily) seen in the previous? Circle them on the diagram and briefly sketch and describe them below.

4. Run Four - Clipping the Bifurcation Diagram

☞ We are going to zoom in one more time, in this case to just the upper arm of the diagram you just plotted; clipping it enlarges it in the final printing. Open the file in Grapher and save it under a different name. Click on the diagram to select it, then right click to PROPERTIES: CLIPPING. Set **Ymin** to 0.75, click ok, save and print.

At 100 plots per r this is not as detailed a diagram as if it were 200 plots per r , but it is enough to see what is going on.

4. EXPERIMENTAL RECORD FOUR – CLIPPING THE BIFURCATION DIAGRAM

Return to the first diagram of the full bifurcation diagram and draw a box around the area this new, clipped diagram represents. Estimate the percent of the detail in the clipped diagram that is *not* visible in the full bifurcation diagram.

% of detail not visible in full diagram

How much longer do you think we could continue enlargements and still see new detail?

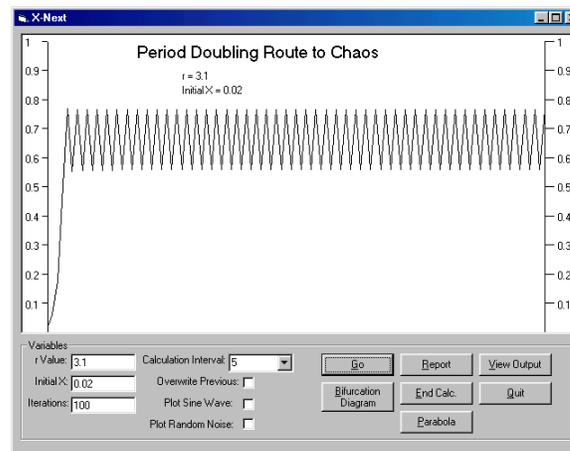
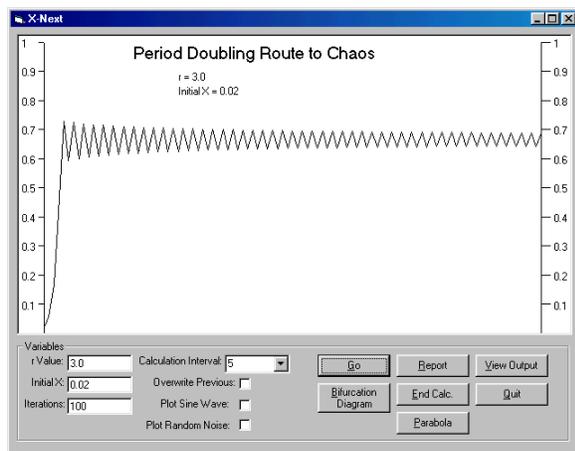
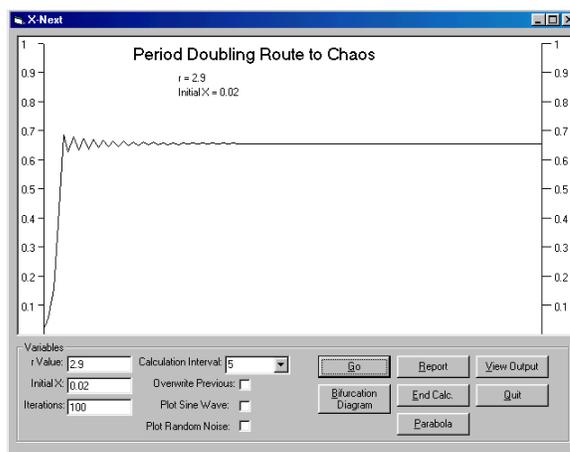
Find and write below some values of r from Experimental Record Four where you think new details would emerge.

Experiment Four - Bifurcation Diagrams Instability and Bifurcations

Recall the X_{next} experiment. At $r = 2.9$ the system attenuates quickly (screen capture to right). At $r = 3.1$ the system oscillates with precision out to all decimal places (screen captures below). The first system is clearly a point attractor (attenuates to one value), and the latter an limit cycle attractor (oscillates between two values). But, what happens in between? At $r = 3.0$ the system never completely stabilizes to a point or an limit cycle attractor.

The implications of this are that as a system approaches a bifurcation it goes from relatively regular behavior, to behavior that becomes unstable, and back to behavior that is regular again.

But is that really what happens, or are we just jumping to a wrong conclusion from limited data, from only r 's of 2.9, 3.0, and 3.1?



More specifically, does the instability occur only as the system approaches a bifurcation, and then quickly stabilize immediately after the bifurcation takes place, or does the zone of instability exist both before and after the bifurcation, or does it do something else entirely?

In this next bifurcation diagram we change the strategy from that used in the previous calculations. Previously, we wanted the system to attenuate as well as it was able in 2000 calculations, and then we plotted the last 50 or 100 iterations to observe its final behavior.

In this experiment it is the early behavior we are interested in, before it has attenuated, especially for low r values. This will tell us how quickly a system attenuates; that is, whether it stabilizes quickly to either a point or oscillatory attractor, or whether it is unstable and takes a long time to settle in.

And since we start with an initial $r = 0.2$, which is relatively low, we want to filter out those first few calculations where the value of X is rising exponentially toward its attractor state.

Run Five - Instability and Bifurcations

- ☞ Create and print a bifurcation diagram with the following parameters
 - ▶ Enter Rmin Rmax **2.40 3.6** (*this captures the 1st and 2nd bifurcations*)
 - ▶ Enter Increment of r **.01** (*calculating fewer r's spreads the data points allowing us to see them better*)
 - ▶ Enter Initial X value **.2**
 - ▶ Enter # of X's per run **25** (*if it attenuates quickly it will happen in these first few iterations; if not we should see a vertical spread of X values*)
 - ▶ Enter # of X's to run (last N of run): **20**
 - ▶ Name and path of Output File **Same as last**
 - ▶ Output to Screen **No**
- ☞ Load data file into Grapher following the instructions above.
- ☞ **PRINT file as a pdf to your folder, then print the pdf to the printer.**

6. EXPERIMENTAL RECORD FIVE – INSTABILITY AND BIFURCATION

Writing Assignment: Examine the bifurcation diagram as it enters and passes through the first and second bifurcations. Describe below how the stability of the system changes as it passes through a bifurcation.

☺ **F** ☺ **i** ☺ **n** ☺ **i** ☺ **s** ☺