CHAPTER 4

Failure Criteria for Soils and Rocks

More Fundamental Concepts

Structural Failure

The two properties that enable a soil or a rock mass to remain in equilibrium when forces are acting to disturb it, for example to resist the gravitational forces produced by its own weight, to sustain the stress concentrations produced around an excavation, or to support the weight of a structure erected upon it, are its shear strength and its tensile strength. If an earth material yields to these applied forces it can only be through the medium of processes of deformation and fracture. If the material is to fail then the cohesion of its mineral and particulate constituents must be overcome, by forces of tension or by pressure acting from within (in which case the fracture surfaces are pulled or else pushed directly apart), or by forces of shear (in which case the fracture surfaces are forced over one another at an angle and against a resistance which depends upon the internal friction characteristics of the earth material in relation to the ambient stress distribution). When we speak of the "strength" of an earth material we mean its ability to resist deformation and fracture by virtue of its properties of cohesion and internal friction. It is these properties that the conventional methods of materials testing are intended to display, under loads that generate forces of shear, tension, and compression.

It is now necessary for us to define some more fundamental concepts, added to those we have already considered.

Friction

Consider a horizontal plane, such as a bedding plane in a rock mass, supporting an elemental block, of weight N, resisting on its surface (see Fig. 4.1(a)). The weight of the block generates an equal and opposite reaction R. N and R together form a compression force normal to the plane of contact, and there is no generated tendency for the rock to move.

If a small horizontal force (not large enough to move the rock) is applied to the block, the reaction R will no longer be normal to the plane of contact. It adjusts in magnitude and direction to equal the resultant N and H. The triangle of forces represents, in magnitude and direction, the relationships between N, H, and R, and the angle θ (see Fig. 4.1(b)).

The normal force $N = R \cos \theta$, and the force H acts across the plane of contact, such that $H = R \sin \theta$.

If the shear force H is increased until the block is just about to slide, R will increase, and so will the angle θ. At the point when sliding begins, the frictional contact holding the materials stable along the plane of contact will be broken, and θ will have attained its maximum possible value, on the surface concerned.

That maximum value is φ, the *angle of friction*, and $\tan \varphi = H/N$ = the *coefficient of friction.*

Referring now to Chapter 2 (Fig. 2.2), where the resultant stress acting along a plane is resolved into a stress σ_n acting normal to the plane and a shear stress τ acting along the plane, if we imagine the plane to be a plane of fracture in the material, movement along that plane in response to the shearing force will be dependent upon the angle of internal friction (φ) of the material concerned, where $\tan \varphi = \tau/\sigma_n$.

Mohr's Circle of Stress

Although the state of stress in a real situation is essentially a three-dimensional problem it is often more convenient, and quite adequate for practical purposes, to simplify the conditions and to picture the problem in two dimensions, in the plane of the intermediate principal stress. The major and minor principal stresses σ_1 and σ_3, acting within a body under load, may then be represented as shown in Fig. 4.2. Consider a plane AD in the material, inclined at an angle θ to the direction of the minor principal stress σ_3.

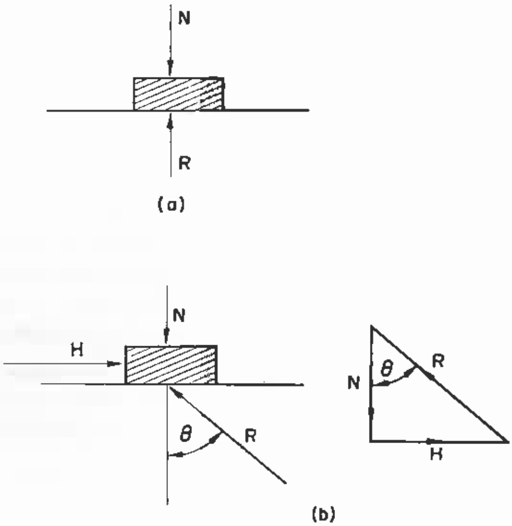


Fig. 4.1. Sliding friction.

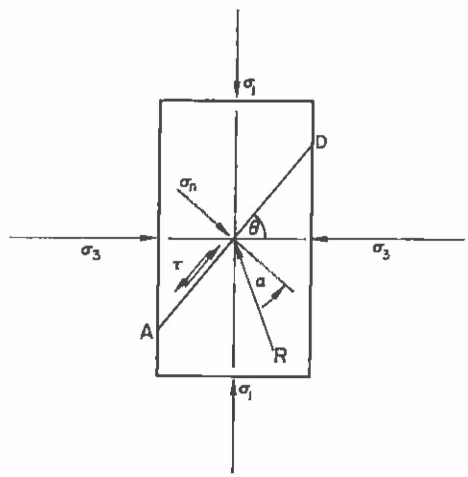


Fig. 4.2. Two-dimensional stress pattern on a plane AD intersecting the minor principal stress direction at an angle θ.

It can be shown that the principal stresses σ_1 and σ_3 induce a stress system in that plane, and we have seen that such a system can be resolved into a shear stress acting along the plane and a normal stress acting across the plane. The magnitudes of these stresses are:

shear stress $$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

and

normal stress $$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta,$$

or $$\sigma_n = \sigma_3 + (\sigma_1 - \sigma_3) \cos^2 \theta.$$

These relationships may be represented graphically by Mohr's circle of stress (see Fig. 4.3).

The following convention is adopted. It is assumed that the direction of the major principal stress is parallel with OY, that is, OX lies in the major principal plane. All principal stresses are then plotted along OX, as are all normal stresses. All shear stresses are plotted along OY.

Procedure

Lay down the axes OX and OY, set off OA and OB along the OX axis, representing σ_3 and σ_1, respectively, to scale.

Construct a circle on the diameter AB. The coordinates of all points on the circumference of this circle represent to scale, shear stresses and normal stresses for all planes passing through the point A, orthogonal to the plane of the diagram.

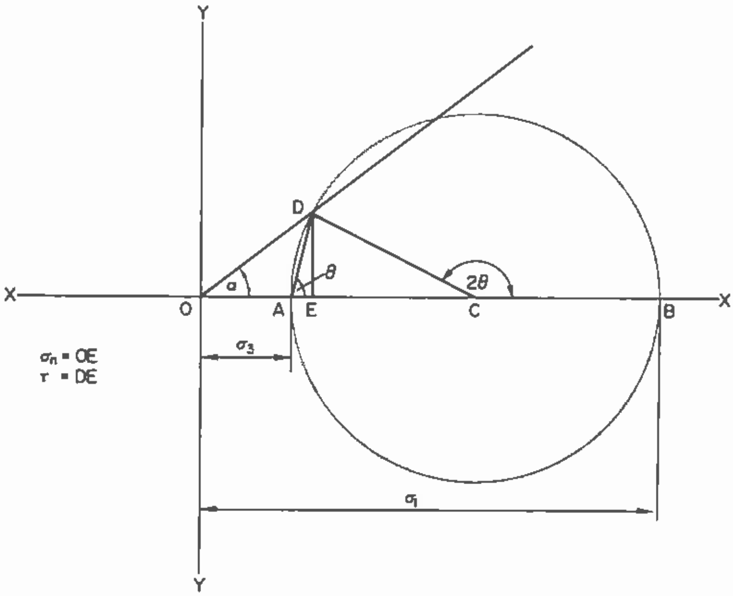


Fig. 4.3. Graphical construction of (Mohr's Circle of Stress), representing the two-dimensional stress pattern of Fig. 4.2.

For example on Fig. 4.2. the plane AD is inclined to the major principal plane at an angle θ. On Fig. 4.3. the line AD, inclined at θ to AB, cuts the perimeter of the circle at D. The coordinates of D then give the normal and shear stresses on the plane AD.

$$\sigma_n = OE,$$

$$\tau = DE$$

Because the normal stress $\sigma_n = OE = OA + AE$

$$= \sigma_3 + AD \cos\theta$$

$$= \sigma_3 + AB \cos^2\theta$$

$$= \sigma_3 + (\sigma_1 - \sigma_3)\cos^2\theta,$$

and the shear stress $= \tau = DE$

$$= DC \sin(180 - 2\theta)$$

$$= DC \sin 2\theta$$

$$= \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta.$$

In Fig. 4.3. the triangle ODE represents the stress situation on the plane AD of Fig. 4.2. The line OD represents the resultant stress R in magnitude, to scale, while the angle DOB represents the angle of obliquity (α) that the resultant stress makes with the direction of the normal to the plane AD.

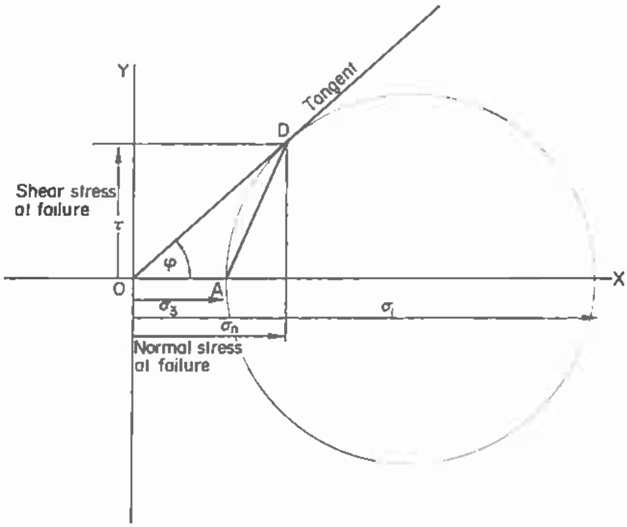


Fig. 4.4. Determination of shear stress and normal stress at failure, using Mohr's circle of stress.

Maximum Shear Strength

The maximum resistance to shear is developed when the angle of obliquity reaches a limiting value ϕ, the angle of internal friction. For this condition the line OD becomes a tangent to the stress circle, inclined at an angle ϕ to the axis OX.

Theories of Failure

Observation on earth materials in the field, in mines and tunnels and in open-pits, tells us that rocks which on some occasions may appear to be hard and brittle may, under other circumstances, display characteristics of plastic flow and creep. The structural geologist sees ample evidence of this in folded strata and in the flow structures often displayed in what now seem to be the most permanent and rigid rock formations. We know too that if an earth material, such as a rock or a soil, becomes highly loaded it may yield to the point of fracture, consequent breakdown, and ultimate collapse.

Failure of Elastic Materials

We may study the processes of deformation and fracture in the materials-testing laboratory, to determine the stress-strain and strain-time characteristics that we have already discussed. Let us now consider what happens when we load our laboratory specimens until they fail.

If the material is elastic and ductile, like a metal such as copper or mild steel, it will display a linear deformation-stress relationship up to the yield point A, shown in Fig. 4.5. It will then deform inelastically over the range AB, as the material becomes strain-hardened, but at B it attains a state of plasticity in which it ceases to resist further deformation, until it ultimately ruptures at the point C.

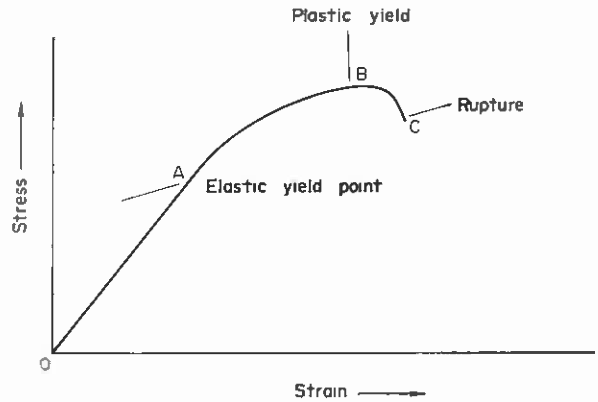


Fig. 4.5. Stress-strain relationship for an elastic-ductile material

If the material is elastic and brittle, like cast iron or glass, it will at first deform elastically under load, but it may then rupture before a discernible yield-point is reached (or very soon thereafter). Failure is liable to be instantaneous and associated with the violent release of energy from the testing mechanism. As a result of this, the specimen may be shattered completely (Fig. 4.6).

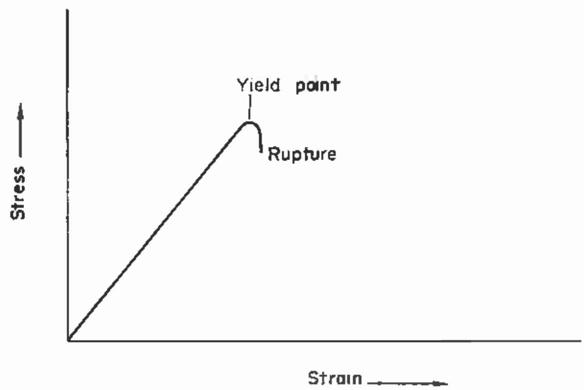


Fig. 4.6. Stress-strain relationship for an elastic-brittle material

Failure of Earth Materials

In general, earth materials such as rocks are not as ductile as the metals, and seldom so brittle as glass, but they display failure characteristics intermediate between the two extremes. The stronger rocks with high silica content and crystalline composition are usually more brittle than the porous sediments, and most rocks appear to have a discernible yield-point before they rupture. If the testing mechanism is suitably designed, to prevent the sudden release of strain energy when the rock begins to yield, the specimen may be observed to retain some resistance to deformation after the yield point (see Fig. 4.7).

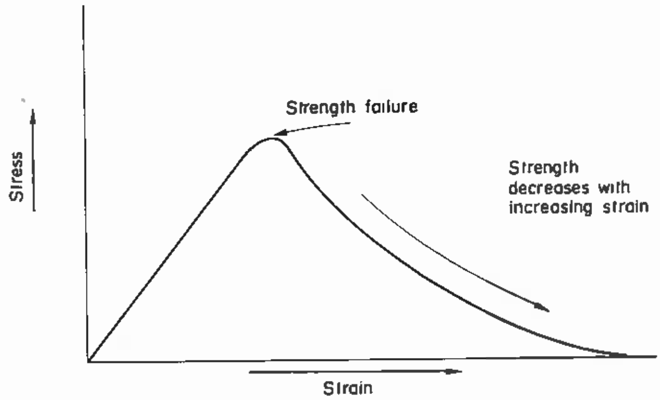


Fig. 4.7. Stress-strain relationship for a rock material loaded on a "stiff" testing machine.

We may therefore interpret the "failure" of a brittle rock material to occur at one or other of:

(a) The attainment of the yield-point.
(b) The point of rupture of the material.
(c) The point at which the material can no longer sustain the imposed load.

However, if the material is not brittle but displays a tendency to deform plastically, the point of failure might be described as:

(d) When the strain rate begins to accelerate under a constant load.

or, if both elastic and plastic deformation characteristics are displayed:

(e) When a maximum allowable percentage of residual permanent deformation occurs in the material.

Criteria of Failure

By selecting one or other of these definitions, and applying it to a specific earth material, we may specify the point at which a given material is likely to "fail" under a given distribution of stress. That is to say, we can establish a "criterion of failure" by means of which the maximum utilizable strength of the material can be identified and used for the purpose of design. The success or failure of our design will then depend upon the extent to which we are correct in our assumptions as to the mode of failure and on the degree to which we can establish the magnitudes and distribution of the stresses and strains involved.

There are many alternative criteria from which to choose, and it is important that we should select one that is appropriate to the particular design problem that we are concerned with. If we were dealing with materials such as metals, or other elastic bodies, we could apply criteria such as the maximum allowable principal stress, or maximum permissible shear stress, or maximum energy of distortion that the material could sustain. Such theories assume that the materials behave perfectly elastically with tensile strength equal to the compressive strength, and they define failure as the beginning of inelastic behavior in the material. We know that earth materials such as soils and rocks, and some constructional materials such as concretes, may behave inelastically under very low stresses and that they have very different compressive and tensile strengths. We should look towards theories of failure which do not involve assumptions that are obviously contrary to our experience. The failure criteria that are most commonly applied in geotechnology include the following:

Mohr's Theory of Failure

Mohr's theory of failure does not attempt to differentiate between failure by deformation or by actual rupture of the material. It states that the failure of a material may be represented by a functional relationship between the shear stress τ acting along the plane of failure, and the normal stress σ_n acting across that plane, such that

$$\tau = f(\sigma_n).$$

The basic assumption here is that the normal stresses, whether they be tensile or compressive contribute towards causing failure, and that shearing stresses also contribute, one being a function of the other. It is not assumed that the material is equally strong both in tension and compression, but it is inferred that in a stress field resolved into three principal stresses $\sigma_1 > \sigma_2 > \sigma_3$, the intermediate principal stress σ_2 has no influence on the failure of the material. The fundamental relationship between τ and σ_n is characteristic of the material concerned, and it must be determined by experimental tests. A graphical representation of the state of stress in the material – the Mohr circle of stress – is used, at the limit condition for failure (Fig. 4.4).

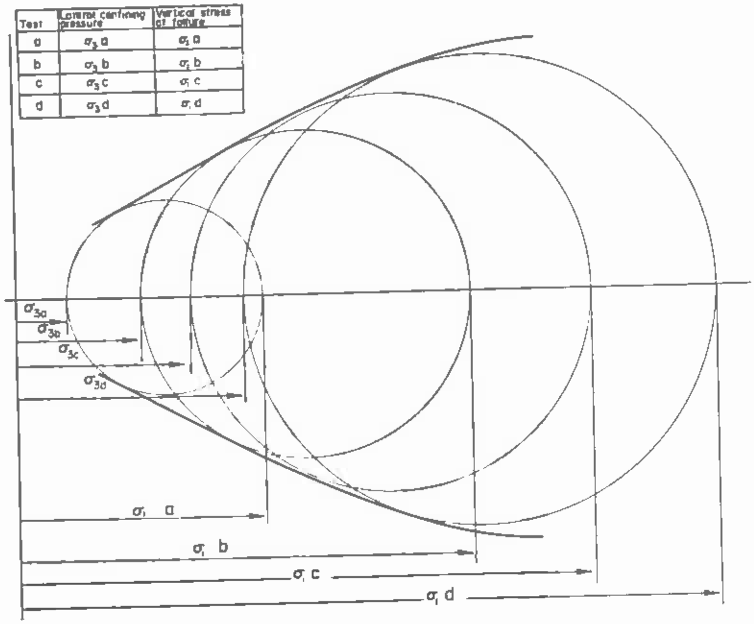
Determination of the Mohr Failure Envelope

A number of drill-core samples of the material are prepared, each with its cylindrical surface enclosed in a flexible jacket. The samples are placed, each in turn, on a triaxial testing machine so that a lateral confining pressure can be applied through the medium of a hydraulic jacket. Keeping the lateral confining pressure constant, the axial load which generates the principal stress σ_1, is increased until the specimen fails. Each test is conducted with a different value of lateral confining pressure, including the unixial ($\sigma_3 = 0$) condition. Sometimes, with rocks, the uniaxial test is also conducted with σ_1 in tension, but this requires special testing arrangements, and it is not easy to perform satisfactorily. With most earth materials, and soils, a tension test is not attempted. The results of each test are plotted as a Mohr circle, so that the complete test produces a number of Mohr circles of stress. The line formed by joining a common tangent to these circles is called the *Mohr Envelope* (Fig. 4.8).

The Mohr envelope defines the conditions for stability of the material under load. If for a given load condition the values of stress lie within the envelope, the material will be stable. But if for a load which produces a certain value of normal stress, the corresponding value of shear stress lies outside the envelope, the material will fail. Since the envelope is symmetrical about the OX axis it is customary to represent the envelope only by the half that lies above the OX axis.

The Coulomb Criterion of Failure for Soils

Referring to the Mohr graphical construction, if the angle of internal friction φ is assumed to be constant for a certain material with no uniaxial tensile strength, then the shear strength



Test	Lateral confining pressure	Vertical stress at failure
a	σ_3 a	σ_1 a
b	σ_3 b	σ_1 b
c	σ_3 c	σ_1 c
d	σ_3 d	σ_1 d

Fig. 4.8. Construction of Mohr failure envelope from the results of triaxial compression tests.

can be represented by two lines passing through the origin O at angles $+\varphi$ and $-\varphi$ to the axis OX (see Fig. 4.9). These lines comprise the Mohr envelope for the material, for which circle A represents stable conditions, circle B represents incipient failure, and circle C represents

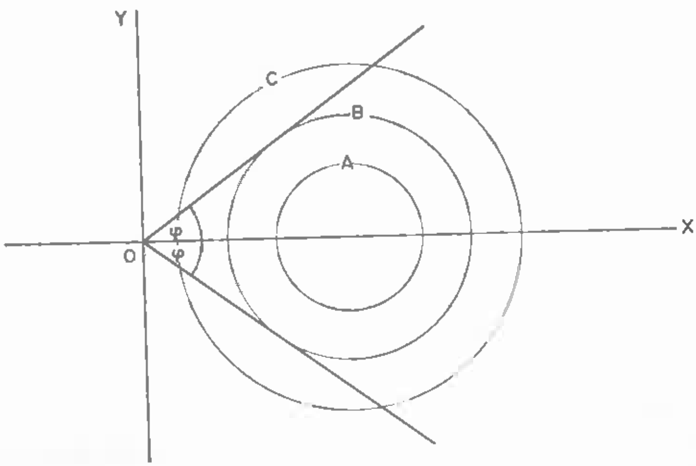


Fig. 4.9. Mohr failure envelope for an earth material with no tensile strength and no cohesion.

stress conditions beyond the limits that the strength of the material can withstand.

Cohesion

A material such as a dry unconsolidated sand will not sustain a slope angle steeper than that which can be held by the frictional contact of the constituent grains. The sides of a trench cut through such a sand will slump to this angle, and the maximum slope of a heap of the sand could not exceed the same angle. Other soil materials, such as damp sands, silts, and clays, will behave differently. The sides of a trench cut through these materials may stand unsupported, or not slump to the "angle of repose" for some considerable time, because the constituent grains are held together by some force additional to that produced by internal friction. This additional force is the *cohesion* of the material.

If a material has no cohesion, but depends entirely upon internal friction for its stability, then its Mohr envelope will intersect the OX axis at the origin O, as shown in Fig. 4.9. But a cohesive material possesses some shear strength even when the normal stress is zero. Its Mohr envelope will therefore intersect the shear stress axis at a finite value, when the normal stress is zero. This intercept on the shear stress axis is a measure of the cohesion of the material (see Fig. 4.10). The resistance to failure exhibited by such a material consists of two parts: (a) internal friction and (b) cohesion.

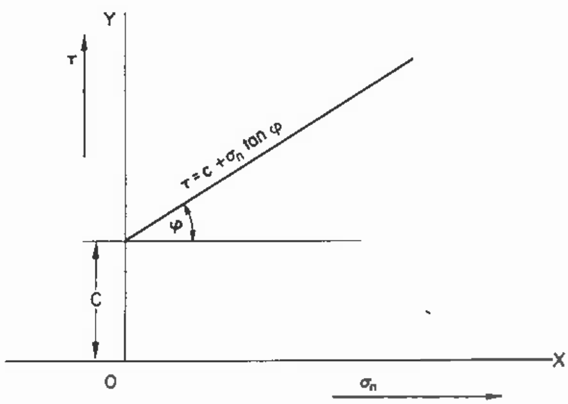


Fig. 4.10. The Coulomb criterion of failure for a soil.

A material such as a soil which is not consolidated can be assumed to possess a constant value of internal friction, no matter what is the value of the normal stress on the plane of shear. Its resistance to shear will, however, also depend on its cohesion and this too is a constant value, independent of the applied stress.

The strength envelope of a soil may thus be described by the straight-line equation

$$\tau = c + \sigma_n \tan \varphi$$

where τ = the shear resistance at failure, i.e. the shear strength,
c = the cohesion,
$\tan \varphi$ = the coefficient of internal friction,
σ_n = the normal stress at failure.

This is the Coulomb criterion of failure— the basic equation in soil mechanics.

Figure 4.10 shows the characteristic Mohr envelope for a cohesive soil. The strength of a soil with no cohesion, such as a dry sand, would be described by $\tau = \sigma_n \tan \varphi$ (Fig. 4.11 (a)), while the strength of a wet clay, incapable of offering frictional resistance to deformation, and subject only to plastic flow controlled by its cohesive power, would have a Mohr envelope similar to that shown in Fig. 4.11(b).

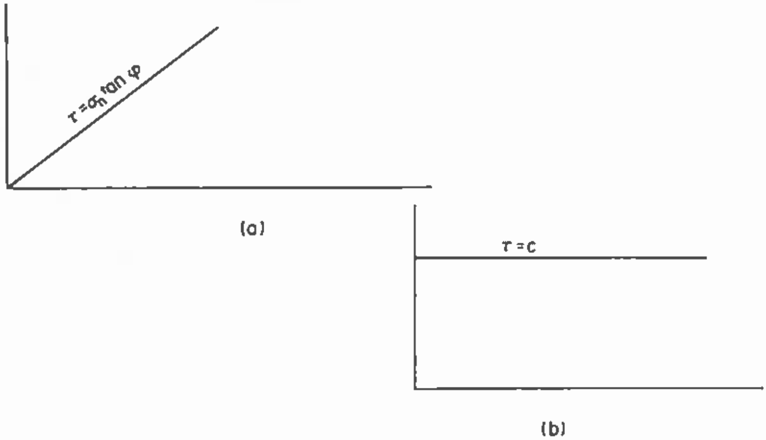


Fig. 4.11. Failure envelopes for soils. (a) Soil with no cohesion. (b) Soil with no frictional resistance to deformation.

The Coulomb-Navier Criterion of Failure for Rocks

The Coulomb criterion may also be applied to brittle solids if it is postulated that on the plane of failure the shear strength is reinforced by a frictional component of resistance to shear. Using the analogy of frictional resistance to sliding of a body resting on an inclined plane, the frictional resistance to shear failure is given by the product of the normal force acting across the plane of failure, and the coefficient of friction along the plane concerned. At the point of failure, when shear sliding is just about to begin, this frictional resistance reaches a maximum value, equal to $\mu\sigma_n$, where μ is the coefficient of internal friction ($\mu = \tan \varphi$).

The Coulomb-Navier failure criterion may then be stated:

At the point of failure the maximum shear resistance of the material (shear strength τ) equals the shear stress on the plane of failure (S_s) plus the internal frictional resistance ($\mu\sigma_n$).

Alternatively,

At the point of failure the shear stress on the plane of failure (S_s) equals the shear strength of the material (τ) minus the internal frictional resistance to shear ($\mu\sigma_n$).

The shear stress and the normal stress on a failure plane inclined at an angle θ to the minor principal stress σ_3, where the major principal stress is σ_1. are:

Normal stress $$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

and

Shear stress $$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

By substituting these values in the Coulomb-Navier equation, the criterion is expressed in a form to define the limiting stress conditions that the material can withstand under tensile and compressive loads. Conversely, if we have data concerning the shear strength, at zero confining pressure, the compressive strength, the tensile strength, and the angle of internal friction, of a particular rock material, it is possible to predict the limiting stress conditions that the material can withstand in terms of the Coulomb-Navier failure envelope.

The method is to plot circles, with radii proportional to compressive strength and tensile strength, respectively, to right and left of zero, on the normal stress axis. Plot the shear strength of the material on the shear stress axis, and insert the appropriate angle of friction by a line tangent to the compression circle. Join up the point and the line so plotted by a smooth curve, to produce the failure envelope (see Fig. 4.12).

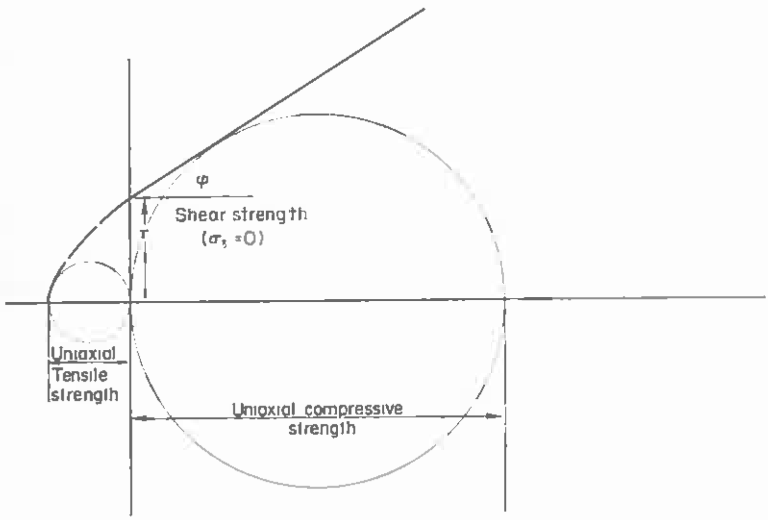


Fig. 4.12. The Coulomb-Navier failure criterion for a rock material.

This envelope may now be applied, in conjunction with the construction of Mohr circles of stress for given principal stress differences $\sigma_1 - \sigma_3$, to estimate the stress magnitudes at which shear failure will occur.

Depending upon the relative magnitudes of the various parameters represented, the envelope so obtained will be more or less curvilinear in the tensile zone, but straight in the compressive zone. This is because the Coulomb-Navier criterion assumes a constant value for the coefficient of friction, except over a limited range of stress at and near the tensile zone. Most rocks have internal friction characteristics that vary with change of loading conditions, so that the angle of friction may decrease with increase of the stress deviator $(\sigma_1 - \sigma_3)$. The failure envelope is then curvilinear in the compression zone as well as in the tensile zone, and it can no longer be described by the Coulomb-Navier criterion.

The Griffith Brittle Failure Criterion

A theory to explain the failure of brittle materials was originally postulated by Griffith in 1924. Griffith worked with glass but his theories have been extended generally to other brittle materials, including rocks. In essence the theory is that fracture of brittle materials is initiated as a result of tensile failure produced by the stress concentrations that exist around the tips of micro-cracks and flaws present in the material. It is assumed that fracture extends from the boundary of an open flaw when the tensile stress on this boundary exceeds the local tensile strength of the material. It can be shown that high tensile stresses occur on the boundary of a suitably orientated elliptical opening, even under compressive stress conditions on the material as a whole.

The theory originally dealt with the mechanism of crack propagation in a uniaxial stress field, but subsequent investigators have extended Griffith's ideas to biaxial and triaxial stress conditions in rocks and also to explain the process of failure around a closed crack.

The results of all this work suggest that in an isotropic rock material, where the orientation of the cracks may be assumed to be random, fracture will occur if the uniaxial tensile strength is less than

$$\frac{-(\sigma_1 - \sigma_3)^2}{8(\sigma_1 + \sigma_3)} \quad \text{(the minus sign denotes tension)}$$

That is, the fracture criterion is

$$\underset{\text{(Tensile strength)}}{S_T} = \frac{-(\sigma_1 - \sigma_3)^2}{8(\sigma_1 + \sigma_3)}.$$

According to this criterion, when $\sigma_3 = 0$, σ_1 becomes the uniaxial compressive strength, and $\sigma_1 = 8S_T$.

Murrell has shown that the Griffith criterion of brittle fracture can be expressed as the equation of a Mohr envelope in which

$$\tau^2 - 4S_T\sigma = 4S_T{}^2.$$

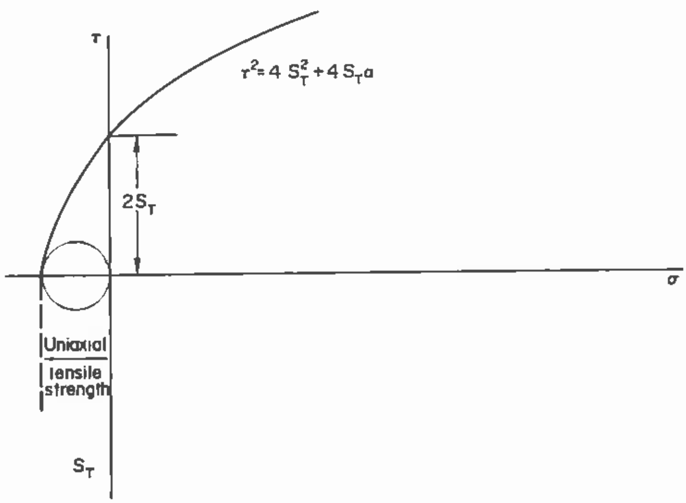


Fig. 4.13. The Griffith failure criterion.

In deriving this criterion it was assumed that the crack retains its elliptical shape until the moment of failure. When the principal stresses σ_1 and σ_3 are tensile, or where the rock is strong and not very highly stressed, this assumption may be valid. But in a weak rock, or in a highly stressed rock containing flat cracks, allowance may have to be made for closure of the cracks before the rock fails. This was done by McClintock and Walsh, by introducing the internal friction coefficient into their analysis. They then derived a modified Griffith criterion for the failure of a closed crack, in the form:

$$4S_T = [(\sigma_1 - \sigma_3)\ (1 + \mu^2)^{1/2}] - \mu(\sigma_1 + \sigma_3).$$

This criterion may be represented by a straight-line Mohr envelope with the characteristic

$$\tau = \mu\sigma + 2S_T$$

which is similar to that of the Coulomb-Navier criterion.

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