CHAPTER 2

The Behavior of Earth Materials under Static Load

Some Fundamental Concepts

In geotechnology we are concerned with the behavior of earth materials under load. The load may originate in various ways – for example, it can be the result of gravitational forces acting upon the mass of the earth material itself and upon the overlying strata. Such loads are important in matters concerning the support of excavations and the control of landslides. Other loads may be generated by tectonic forces acting upon the Earth's crust, as evidenced by geological faulting and folding, or by inherent forces within the mineralogical and petrofabric structure of a rock material, as evidenced by crystal twinning, cleavage and foliation. Yet another source of load comes from the pressure generated by fluids acting within the pores and interstices of the earth material. All these forces are primary, that is, they exist before any engineering or excavation is done, and they are withstood by the inherent strength properties of the soils and rocks concerned.

The creation of an excavation, or the erection of a structure on a soil or rock foundation, in an earth material at an original state of equilibrium, disturbs the original force field and a new, redistributed, force field is produced. In the case of an excavation, the earth or rock walls tend to move and they try to fill the excavation from all sides, while a structural foundation imposes added loads upon the soils and rocks on which it stands. Those soils and rocks then tend to yield, from regions of high pressure to regions of lower pressure. If a condition of equilibrium and stability is to be maintained, the inherent strength properties of the earth materials must also withstand the added loads, and resist these generated tendencies to yield. It is this secondary or generated force field that must be controlled in underground support systems, in the control of caving and ground subsidence, in the stabilization of earth and rock slopes, and in foundation engineering.

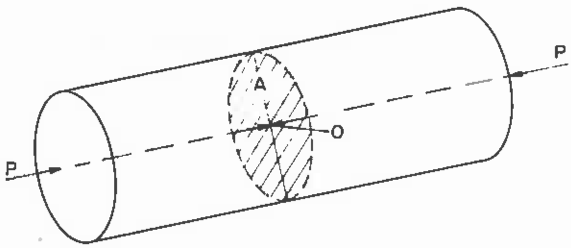
Another form of secondary force field is that generated by mining tools such as rock cutters and picks, drills, explosives, and blasting materials, which are applied to break rock in the process of excavation.

The response of an earth material to these forces is described in terms of *load*, *stress*, and *strain*. The load at a point within a rock mass, being the resultant of all the forces acting upon the material at that point, is sometimes termed the *strata pressure*.

Stress

The resultant force induces a state of stress in the earth material. Stress may be defined as the force per unit area. The concept involves consideration of a plane cutting through an element of the material concerned, on which the force P acts (Fig. 2.1). Since in nature the application of a force necessarily involves the generation of an equal and opposite reaction, the force can be either compression or tension, depending upon whether it is directed towards, or away from, the plane considered.

The force P produces a uniform stress $\sigma = P/A$ acting on a normal cross-section of area A, and the concept of stress acting at a point is the limiting value of $\delta P/\delta A$ as A approaches zero at the point concerned.



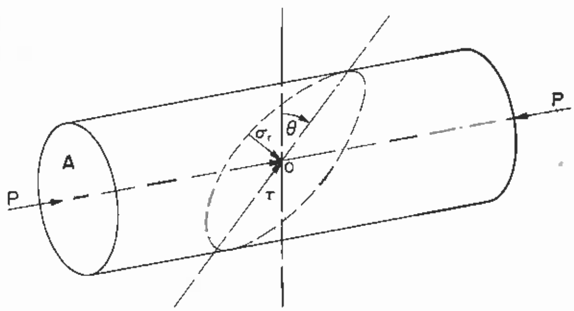
Stress on plane A = $\sigma = {}^{P}/_{A}$

Stress at point O = ${}^{\delta P}/_{\delta A}$ at limit when $A \rightarrow$ zero

Fig. 2.1. Stress at a point

Normal and shear stress components

If in Fig. 2.1, we imagine the cross-section to be inclined at an angle θ to the normal, as shown in Fig. 2.2, the area of this cross-section is $A/\cos\theta$.



Normal stress σ_n and shear stress τ, components of stress at point 0.

Fig. 2.2. Normal and shear components of stress

The force P can be resolved into two components, P_n acting normal to the cross section of area $A/\cos\theta$, and P_s acting parallel with the surface of the section, where $P_n = P\cos\theta$ and $P_s = P\sin\theta$.

The area of the inclined section, to which P_n is normal, is $A/\cos\theta$.

Therefore the normal stress $P_n = P/A.\cos^2\theta$.
And the surface shear stress $P_s = P/A.\sin\theta\cos\theta$.
Hence, at the limit, at the point 0,
normal stress $\sigma_n = \cos^2\theta$
and shear stress $\tau = \sin\theta\cos\theta$.

Principal stresses

Considering all possible values of θ in Fig. 2.2, corresponding to various angles at which the cross-section is taken, maximum and minimum values of σ_n occur when $\theta = 0°$ and $90°$, for which angles τ = zero. The stresses σ when $\theta = 0°$ and $90°$ are termed principal stresses and the planes inclined at 0° and 90° are principal planes. On these principal planes $\tau = 0$, i.e. no shear stress exists.

Maximum values of τ occur when $\theta = 45°$, i.e. along planes inclined at 45° to the principal planes.

The principal stresses in two dimensions are usually denoted by σ_1 and σ_2 where $\sigma_1 > \sigma_2$. For convenience in notation compressive stresses are usually regarded as positive, and tension as negative.

Stress in three dimensions

The state of stress in a material is essentially a three-dimensional problem. In Fig. 2.1 and 2.2 the magnitude of the resultant stress P and the direction in which it acts are known. If that were not so, the resultant stress magnitude and direction could be deduced if three mutually perpendicular normal stresses (principal stresses) and the corresponding principal planes were known. Conversely, the resultant stress at a point can be resolved into three mutually perpendicular principal stresses σ_1, σ_2, and σ_3, acting at that point, where $\sigma_1 > \sigma_2 > \sigma_3$.

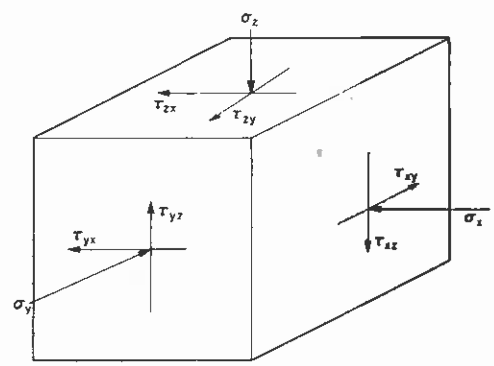


Fig. 2.3. Components of stress on an elemental cube of material
(The stresses on those sides of the cube not shown are equal and opposite to those shown on this view.)

In the general case the resultant state of stress may be represented as shown in Fig. 2.3, where the three mutually perpendicular axes x, y, and z, do not coincide with the principal stress directions. Hence the faces of the elemental cube are not parallel to the principal planes of stress and so they experience finite values of shear stress. The resultant stress at the point of intersection of the axes x, y, and z can then be represented by those on the elemental cube at the limit, when the side length of the cube approaches zero. It can be seen that this resultant is represented by nine stress components

$$\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \sigma_y & \tau_{yx} & \tau_{yz} \\ \sigma_z & \tau_{zx} & \tau_{zy} \end{matrix}$$

involving six quantities, since (because the cube is in equilibrium and therefore there is no turning moment on any of its faces)

$$\tau_{xy} = \tau_{yz},$$
$$\tau_{xz} = \tau_{zx},$$

and

$$\tau_{yz} = \tau_{zy}.$$

Strain

The effect of stress on an earth material is to tend to produce deformation, that is, to produce change in length, change in volume, or a change in shape. This deformation is measured in terms of strain, which is a non-dimensional quantity, represented by the ratios:

$$\text{Longitudinal strain } \left(\frac{\text{change of linear dimension}}{\text{original length}}\right) = \delta l/l$$

$$\text{Volumetric strain } \left(\frac{\text{change of volume}}{\text{original volume}}\right) = \delta v/v$$

$$\text{Shear strain } \left(\frac{\text{angular displacement}}{\text{length}}\right) = \frac{d}{l} = \tan \varphi.$$

Infinitesimal strain

Classical theory, concerning the behavior of structural materials under load, assumes that the materials are elastic, and it describes the stress-strain relationships at the onset of deformation, when the strains are infinitely small. For example, the concept of shear strain in Fig. 2.4 assumes that the angle φ remains constant throughout the thickness of the material. This will only be approximately true, and for small displacements.

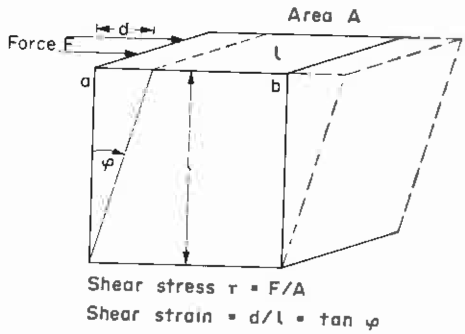


Fig. 2.4. Shear strain.

Finite strain

If the strains are appreciable then the angle φ will not be constant throughout the material, so that the boundaries of the deformed rhomboid will take a curved form. In Fig. 2.5 an element in one plane of the material is pictured to have a circular shape when unstrained, but in the strained condition the circle becomes an ellipse.

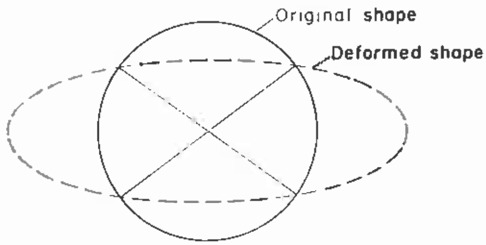


Fig. 2.5. The strain ellipse.

The strain ellipsoid

In three dimensions the ellipse becomes an ellipsoid. The shear strain in a given direction is then measured in terms of the angle between the radius vector and the tangent to the strain ellipsoid at that point.

Pure shear and simple shear

If the strain ellipsoid results from a deformation mechanism in which both the maximum shear planes have been active, then the material is said to be in a state of pure shear (Fig. 2.6). The type of shear mechanism represented in Fig. 2.4 is known as simple shear, or simple rotational shear, in which the active shear planes on which deformation occurs are assumed to be either parallel or concentric.

Heterogeneous and homogeneous strain

The geological processes of rock deformation can be seen to have produced contorted strata from what were once regular and uniform deposits. In the course of this change, elements of the material that were originally spherical become irregular in shape, and boundaries that were once linear become curvilinear. This complicated pattern of observed strain is heterogeneous strain, and as an aid towards deduction of the processes that have contributed to what he sees, the structural geologist may sometimes consider the total heterogeneous strain to be composed of elements in which the strain is homogeneous. In such elements circles become ellipses and spheres become ellipsoids.

In the general case a strain ellipse may contain components of distortion, dilation, and rotation. The deduction of these components, in three dimensions, from the evidence presented in a rock exposure, can be very complex. Probably the most easily interpreted evidence comes from the mineral inclusions or spots in such rocks as slate. These inclusions often have a discolored oxidation zone around them which, before the rock was deformed, would be spherical around the inclusion, but now in the metamorphosed slate it is ellipsoidal. Similarly, once-spherical pebbles or oolites in a sediment may now appear as ellipsoidal constituents of a deformed rock. In these cases the end-product of deformation, the ellipsoid, can be used to provide a direct geometrical description of the resultant pattern of strain in the rock, but it can provide no information as to the relative contributions made by distortion, dilation, and rotation, in the process.

In this connection, some useful information can sometimes be gained from fossils, particularly if these are fossil shells that are known to have been originally symmetrical in shape. The symmetry of such a shell is changed by the rock deformation process, and each shell now provides a graphic picture of the strain to which it has been subjected. By collating the evidence provided by a number of such shells in a rock mass the resultant strain ellipse in the mass may be deduced.

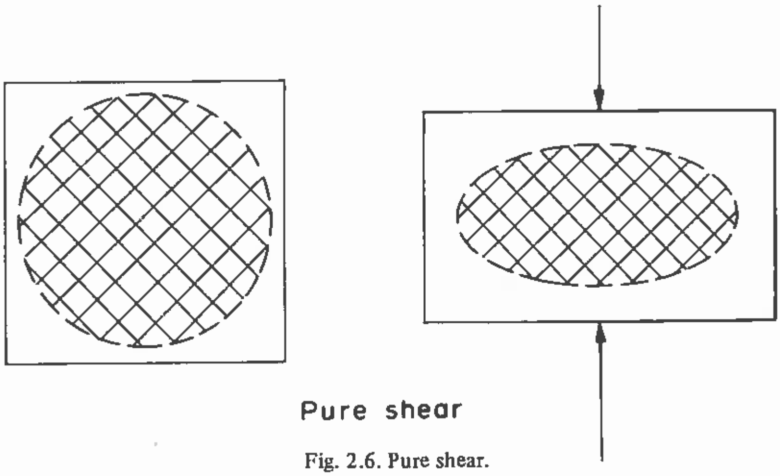


Fig. 2.6. Pure shear.

Plane Stress

When studying the distribution of stress and strain in a structure it is often convenient to consider a two-dimensional model, in one plane. Sometimes we are concerned only with the conditions on the surface of a solid structure such as the skin stresses on the wall of an excavation. In these cases it is assumed that the normal and shear stresses on the plane concerned are zero. This is the plane stress condition. Such an assumption would be justified if the stress situation was being pictured as one acting on the surface of a structure whose lateral dimensions, in the plane being considered, are large relative to the thickness normal to that plane. The assumption implies that there is no constraint in a direction normal to the plane of stress.

Plane Strain

If the stress situation is one acting within a solid structure, a plane section taken at some position within the structure might be chosen for study. In this case it may be more appropriate to assume conditions of plane strain, and to assume that there is no deformation normal to the plane under consideration. That is, the material is completely restrained in directions normal to the plane of measurement.

We may imagine conditions of plane strain to occur on the cross-section of a rock core cylinder, at a position remote from its ends. Another example would be the perimeter of a mine shaft or a tunnel underground. In both cases we would be justified in assuming complete restraint in a longitudinal direction, normal to the cross-section, and deformation is only possible in the plane of the section.

Springs, Dashpots, and Weights

There are two general methods of approach when studying the behavior of earth materials under load. The first is to devise methods of mechanical testing on representative samples of the soils and rocks. The tests may be conducted in the laboratory, and sometimes also in the field. In either case the results observed will be determined partly by the inherent properties of the earth material, partly by the characteristics of the testing equipment, and partly by the method followed in carrying out any particular test. The observer must be very careful in trying to establish what really are the properties of the earth material when assessing the results from such tests.

The second basic approach is to consider the behavior of imaginary materials having specific physical properties that can be precisely stated in mathematical terms. One creates, in fact, mathematical models of various types of material. The two approaches come together in that the mechanical behavior of real materials under load must be compared with the mathematical behavior of the imaginary materials. The engineer uses his mathematical models to help him forecast how real materials will behave when they are subjected to the loads imposed by the structures he designs. The degree to which his design estimates will approach a real situation in geotechnology depends upon the extent to which the behavior of earth materials approaches that of his models.

It is not uncommon for researchers in geotechnology to become so absorbed in the behavior of their models that, for them, the models assume the importance of reality. In consequence, theoretical approaches to soil and rock mechanics grow evermore complex and elaborate, while practical application of the theories lags behind. Indeed, the engineer in the field is sometimes heard to complain that the theoretical models are more difficult for him to understand than are the rocks and soils they are supposed to represent. Were it not for computers much of the theoretical structure would not exist, because the mathematical treatment would otherwise often be impracticable of solution. But the solutions yielded by computers are only to be believed in relation to the veracity of the information that is fed into them, and the engineering factors and the physical properties of rocks and soils, as they exist *in situ* are as yet obscure, to say the least.

Some of the difficulties encountered when trying to establish the engineering data required for feeding into the equations are due to the practical problems of achieving controlled conditions in the field. It is much easier to do this in the laboratory. Hence there has evolved a third line of approach in geotechnology, based on laboratory studies of physical models constructed of simulated, or sometimes "equivalent" materials. Here the behavior of the material model forms an intermediate step when attempting to correlate the mathematical model with a real situation. With these words of warning in mind, let us now look at some properties of ideal materials.

Elastic Materials

A perfectly elastic material can be imagined to behave, in response to an applied load, as does a spring, in that it responds immediately to the load. It displays an instantaneous change in dimension that is directly proportional to the stress generated by that load.

Young's modulus

In such a material the relationship stress/strain is constant. In terms of linear dimension the ratio is termed "Young's Modulus of Elasticity" (E),

$$E = \sigma/\epsilon$$

The greater the value of E for a material, the less will be the deformation produced by a given value of stress, and the stronger the material will be. In an ideal elastic material E will be the same both in tension and compression, but in a real material this may not necessarily be so. Real materials may also display non-linear elasticity, in which, although the stress-strain path may be the same both for unloading and loading, the path is a curvilinear one, with different values of E at different levels of stress. An isotropic material displays the same value of E in all directions, at a given value of stress, but in an anisotropic material E differs in different directions through the material.

Bulk modulus

An elastic material subjected to a uniform pressure from all directions would experience a hydrostatic tensile or compressive stress, and its volume would be affected by any change in the applied pressure. The extent of that change is dependent on the *Bulk Modulus* or *Compressibility* of the material.

$$\text{Bulk modulus } (K) = \text{Original volume}\left(\frac{\text{Change in pressure}}{\text{Change in volume}}\right),$$

$$K = V\left(\frac{\delta P}{\delta V}\right).$$

Modulus of rigidity

The change of shape of an elastic material subjected to a shearing force is determined by its *Modulus of Rigidity* or *Shear Modulus*.

In Fig. 2.4 consider the cube *abcd* fixed along the plane *cd* and of base area A. Suppose a force F to act in the plane *ab*, producing a shear stress $F/A = \tau$. This distorts the cube by a shear strain φ. The modulus of rigidity of the material is G, where

$$G = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\varphi}$$

Poisson's Ratio

Consider an element of an elastic body, say in the shape of a prism or a cylinder, and suppose this to be loaded, either in tension or compression, then the strain in the axial direction will be accompanied by strain in a transverse direction. If the applied load generates a tensile stress axially then the lateral strain is a contraction, whereas if the axial load generates an axial compression then the lateral strain is an elongation.

$$\text{The ratio } \frac{\text{lateral strain}}{\text{longitudinal strain}} = \text{Poisson's ratio } (\upsilon).$$

It is incorrect to express this change in shape in terms of the generation of lateral stress in response to and of opposite sign to the axial stress, if the material is not constrained. In such a case, if the material is uniaxially loaded the stress throughout the specimen will be either tension or compression in the axial direction. But if the material is constrained, and the element of the material is confined by other elements surrounding it, so that it is not free to deform laterally, then an axial compressive stress will generate lateral compressive stresses, the magnitude of which will depend upon Poisson's ratio for the material and the degree of constraint. The three elastic moduli, Young's modulus E, the Shear modulus G, and Poisson's ratio υ, are interdependent,

$$G = \frac{E}{2(1 + \upsilon)}.$$

Rheological Properties of Earth Materials

Real earth materials do not display ideal elastic behavior, although some may approximate to it over a limited range of conditions. An elastic relationship between a stress resulting from applied load and the observed strain implies that the internal forces that hold the atomic and molecular structure of a solid material together are not overcome, so that they are able to bring the material back to its original shape and size when the applied load is removed. While in an ideal elastic material recovery is instantaneous upon relief of load, some materials may display delayed, although complete, elastic recovery.

This only happens if the stresses set up by the applied load are less than critical value, called the "*yield point*" of the material. If the applied stresses are higher than those representing the yield point then the internal elastic forces are overcome. The individual atoms and molecules of the material will then suffer permanent dislocation. Such phenomena occur within the crystal structure of the mineral constituents of crystalline rocks, but they also occur between and within the fragmented rock materials and cementing matrices of consolidated sediments, in broken rock masses, and between the granular constituents of soils. In the latter case the bond strength between the constituents is so low that yield occurs at very low values of stress, when compared with the sedimentary and crystalline rocks.

A soil is a three-phase system, composed of solid matter, water, and air. Rocks, too, are essentially multiphase materials in which internal fluid pore pressures play an important part in determining the material's response to stress. The more porous the material the more significant will be the effects of pore fluids and pore fluid pressures in lowering the respective stress levels at which occur the yield point, the onset of time-dependent deformation, and the permanent dislocation of constituent boundaries.

Rheological Models

The behaviour of materials that are not ideally elastic can be studied in terms of the behavior of rheological models. These models may be imagined as being composed of spring, dashpot, and friction elements, in various modes of combination. The spring element has elastic properties, while the dashpot is viscous and the friction element displays plastic flow (see Fig. 2.7).

The elastic element displays a constant relation between stress and strain, defined as Young's modulus E, such that

$$\sigma = E.\epsilon.$$

The viscous element displays a time-dependent relation between stress and strain, and a linear relation between stress and the rate of strain, such that

$$\sigma/\eta = d\epsilon/dt$$

where η is the coefficient of viscosity.

These elements may be combined to form models, the behavior of which can be described by linear equations, as shown in Fig. 2.7.

No	Model	Characteristic	Name of investigator
1		Ideally elastic body	Hook
2		Linear liquid	Newton
3		Elastic and viscous body	Kelvin Voigt
4		Elastic liquid	Maxwell
5		Elastic and viscous body	Zener
6		Elastic liquid	Burgers

Fig. 2.7. Elements of linear rheological models.

No	Model	Characteristic	Name of investigator
1		Ideally plastic body	St Venant
2		Plastic body	Kepes
3		Strain resistance	—
4		Velocity resistance	—
5		Plastic liquid	Bingham
6		Elastic and plastic liquid	Schwedoff
7		Elastic and plastic liquid	Prager

Fig. 2.8. Elements of non-linear rheological models.

The ideal plastic element displays no yield until the stress reaches a critical value, after which there is a constant rate of strain. This process is modeled as a dry friction sliding or St. Venant body. A more general plastic solid model is the Kepes model, in which the magnitude of the yield point is a function of strain. The Bingham, Schwedoff, and Prager bodies are other non-linear models, the first representing a plastic liquid, the second an elastic-plastic liquid, and the third an elastic-plastic liquid exhibiting delayed elastic recovery when the load is removed (Fig. 2.8).

Elastic Behavior in Real Earth Materials

When we examine the behavior of real earth materials we find that, under static loading conditions, only the hardest and dense, crystalline, non-porous igneous and metamorphic rocks approach ideal elastic behavior. The generalized stress-strain relationship for a rock is more likely to take a curvilinear form, as shown in Fig. 2.9. With increase of load from a zero condition the slope of the stress-strain curve increases as the pores and interstices of the material are closed by the applied pressure. Only after this will a rock material display approximately

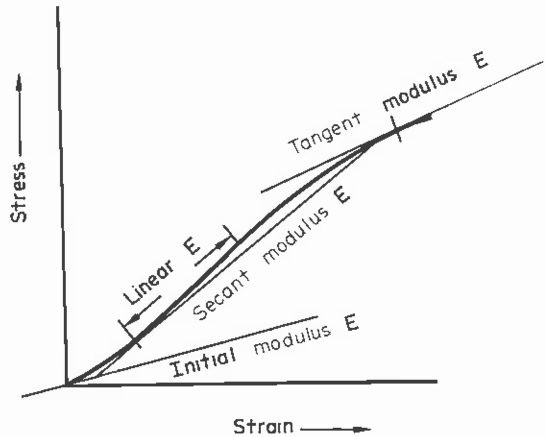


Fig. 2.9. Generalized stress-strain curve for a rock (up to the yield point).

linear elastic response and this continues until the onset of internal slip, dislocation, and microfracture within and among the rock constituents heralds the approaching yield point. The approximate linear response changes to a curvilinear relation, and the slope of the stress-strain curve diminishes after the yield point. The magnitude of the Modulus of Elasticity thus varies from a minimum at zero load to a maximum somewhere between zero load and the yield point.

We should therefore specify to what point on the stress-strain curve we are referring, when quoting the numerical value of Young's modulus for a rock material, and we should choose the most appropriate figure from a range of values to fit the circumstances involved in a particular engineering problem. For example, if we are dealing with dynamic phenomena such as occur in rock drilling or in seismic work, it is the initial dynamic modulus that we will be concerned with, but a strata control or excavation support problem will have to deal with the response of the rock over a range of stress conditions and we will be concerned with an average value over that range, in which case we want the secant modulus. Or perhaps we will be concerned with a specific value of the modulus at a particular stress level – the tangent modulus at that value of stress.

It should be pointed out here that the initial dynamic modulus will, in all probability, not be the minimum value of Young's modulus for the material, but is more likely to be near the maximum value, because the stress-strain curve under dynamic loads usually takes a convex-upwards form (see Fig. 8.16). The stress-strain relationship for an individual rock may differ from that shown on the generalized curve, depending upon its special characteristics. Hendron quotes the curves shown in Fig. 2.10, in which the type I curve applies to basalts, quartzite, diabase, dolomites and strong massive limestones, all of which approximate to elastic behavior. The type II curve applies to softer limestones, siltstone, and tuff. This curve displays a decreasing E value from an initial maximum, due to high strength plastic yield, and decreases in strength as failure is approached. The type III curve begins with an initial minimum E-value, which increases as the material consolidates slightly under load, and the initial plastic phase is followed by the major elastic phase. This would apply to fine, massive, sandstones and some granites. The type IV curve shows a transition to plastic deformation as failure is approached

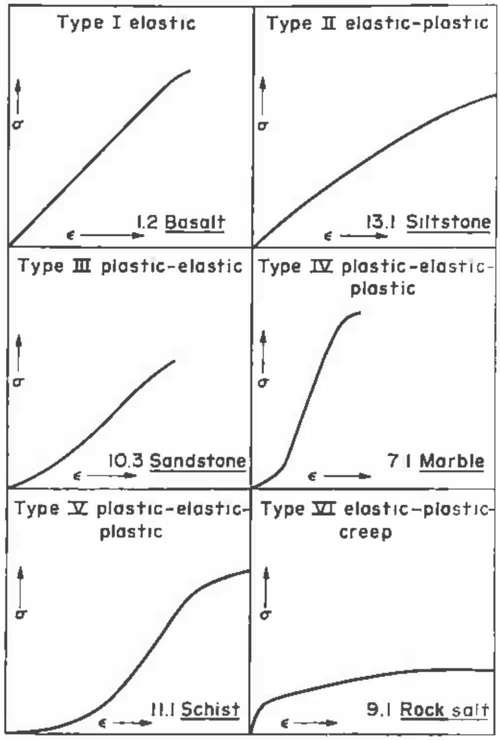


Fig. 2.10. Typical stress-strain curves for rock in uniaxial compression (Hendron, after Miller)

typical of fine metamorphics, marble and gneiss. Type V includes the porous metamorphics and sediments, and type VI the saline evaporite rocks.

Rock hysteresis

On relief of load the strain-stress relationship of a linearly elastic rock will follow that displayed on loading. Any departure from linearity and ideal elasticity shows up as a "hysteresis loop" on load cycling. If the elastic recovery is not instantaneous some residual strain will remain on relief of load. That part of the residual elastic strain recovered in the course of time is "*delayed elastic recovery*". Should there be any plastic deformation on increase of load this will not be recovered on unloading and the width of the hysteresis loop will be increased, as a result. The effect of load cycling with progressively increasing increments of load is shown in Fig. 2.11(a) for a granite and in Fig. 2.11(b) for a porous sediment.

Stress-strain relationships in soils

Stress-strain relationships in unconsolidated earth materials in the form of assemblages of fractured rock and soil, are important in many aspects of geotechnology. The justification for applying elastic design theory to earth materials in general is that after several load-unloading cycles, either during large-scale *in-situ* tests or during laboratory tests, an identifiable stress-

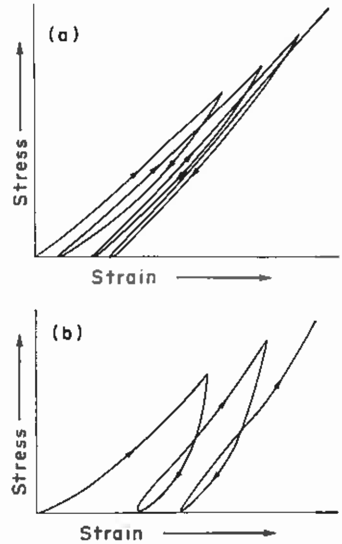


Fig. 2.11. Stress-strain curves during load cycling: (a) on a fine-grained igneous rock, (b) on a porous sedimentary rock.

strain relation usually becomes apparent and this, although seldom linear over the whole range of observation, can often be approximated to a linear relation over a limited range of load. Hence even a non-linear earth material such as a soil can be approached by elastic design theory if the "*deformation modulus*" for the particular stress distribution can be identified.

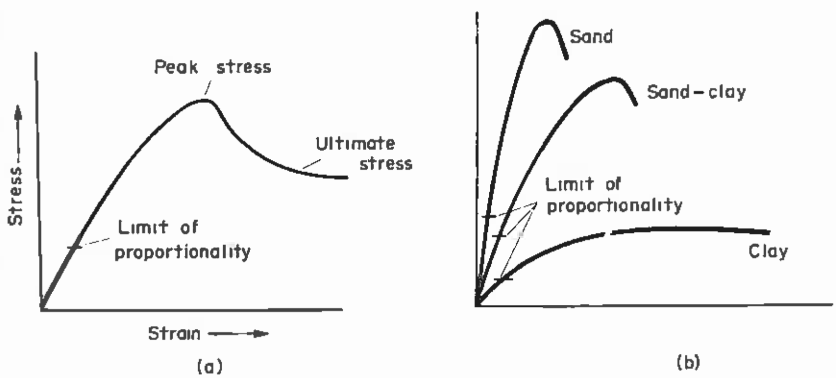


Fig. 2.12. (a) General stress-strain relationship for a soil. (b) Typical stress-strain relationships for soils on initial loading in compression.

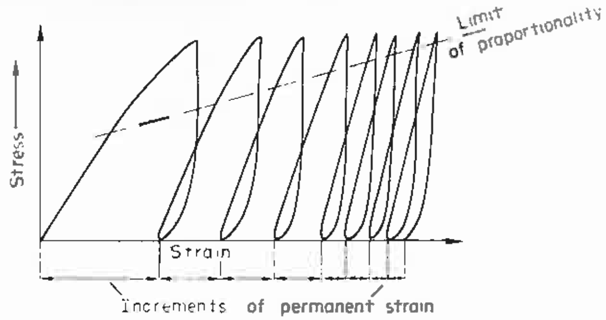


Fig. 2.13. Stress-strain curves during load cycling on a sandy-clay soil

Typical stress-strain relationships for a single application of compressive load to various soils are shown in Fig. 2.12. Soils containing clay often display linear proportionality over a limited extent from zero load, but most of the strain is not recoverable on unloading. A typical load cycling sequence is shown in Fig. 2.13, from which it can be seen that the deformation modulus and the limit of proportionality increase with each progressive cycle, while the increment of permanent strain per cycle diminishes. In the early sequences the increment of permanent strain is large, but it becomes progressively less per cycle as the material is made more dense by the closure of interstices and pores under load. As load-cycling progresses further, either the hysteresis loop will ultimately close (and the material will then behave elastically over that range of stress), or the increment of residual strain per cycle will attain a constant value (and the material will be behaving plastically).

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