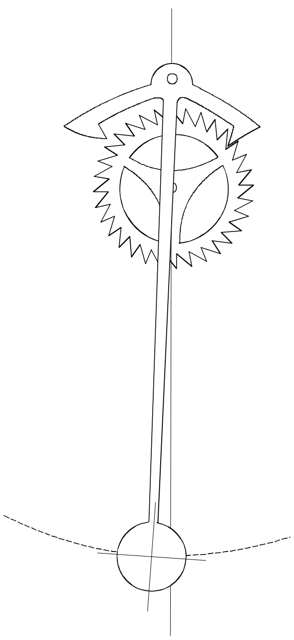
Laboratory Instruction-Record Pages

The Driven Pendulum

Geology 200 - Evolutionary Systems
James Madison University
Lynn S. Fichter and Steven J. Baedke

Brief History of Swinging

Many things in this universe oscillate - alternate between two or more states; pendulums, electronic circuits, biochemical reactions, etc. Of these the pendulum is the most basic and universal. Deciphered by Galileo (1564-1642) the behavior of a pendulum has served as the quintessential limit cycle attractor, the model for all oscillatory systems.



Study of the pendulum began when Galileo observed the swinging of a chandelier hanging in a cathedral. From his studies Galileo concluded that the swing has a constant period—that is, the period of each pendulum is independent of the size of the arc through which it passes, a "fact" later demonstrated not strictly correct[1]. Today we know that the period of the pendulum remains constant as long as the pendulum's angle is no greater than about 20 degrees, and even then, it is not completely precise.

Nonetheless, based on these observations Galileo began to use a pendulum as a stopwatch to time his experiments, but never found a way to convert it into a practical clock. It was Christian Huygens of the Netherlands in 1657 who produced the first driven pendulum clock based on Galileo's principles. Until the beginning of the twentieth century, pendulum clocks were the most precise available, and because of this the average person assumes that the pendulum exhibits regular, precise, periodic behavior—the ticktock of a Grandfather clock. However, they do not.

Horology is the science of timekeeping, clocks, and watches, and even today has many amateur adherents. The goal always has been to devise a mechanism that is reliable and extremely accurate. However, the closer clock makers got to the goal the more elusive it seemed to become because there always remained some perturbations. Horologists refer to the small variations in a pendulum's swing as *flicker noise*, and it has been well studied over the years in attempts to eliminate it. For example, the materials from which pendulums are made change size with temperature, affecting the period, while differences in air pressure and humidity change the air resistence on the pendulum, affecting its period. So, minimal-size-changing metals were concocted, and pendulums were placed in hermitically sealed chambers. And although precision improved, complete precision remains elusive; real world pendulums are subject to too many sensitive dependent conditions.

Flicker noise, however, is also known as *pink noise*, or *one-over-f noise*, meaning the swing of a pendulum is, in fact, a chaotic phenomena, which is why it is impossible to control. So, even in a clock

[1] A pendulum moving along a greater arc traverses a greater distance and its velocity is greater, for it falls from a greater height and at a more acute angle. As a result of these factors, its speed is far greater. The surprising conclusion - the pendulum traverses a longer distance in a shorter time, than in a shorter distance, and its period is shorter.

pendulum which seems well behaved there is sensitive dependence, dependent on minor variations in material, friction, temperature, air pressure, etc.

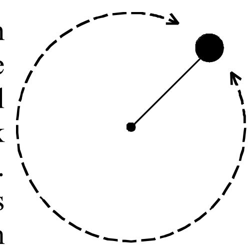
The computer experiments here are based on the equations describing a driven pendulum. They are just models of a possible real world pendulum, and as such can be made to behave as a true limit cycle attractor. However, even in the math world a pendulum is not always well behaved. Increase the driving energy, increase the frequency of the driving force, decrease the damping force (friction) and errors begin to creep in, leading to period doubling, and eventually chaos. Plus, all of these are sensitive dependent on the initial conditions.

Pendulum behavior is thus analogous to the X_{next} (logistic) system; at high 'r' deterministic—but sensitive dependent, and unpredictable. Which means that in a real world with real materials and real conditions their behavior is likely unpredictable over an even larger range of variables.

And, what is true of the pendulum is true of all oscillating systems governed by positive and negative feedback—pretty much the entire universe—deterministic, sensitive dependent, and unpredictable.

The Driven Pendulum

A ***driven pendulum*** is one in which at the end of each swing the pendulum is given a "kick", a nudge or push, to keep it going. If the kick exactly equals the energy of the pendulum then it will swing in a regular pattern. Greater kick and its swings will increase; lesser kick and its swings will diminish. This is the way a Grandfather clock works, the kick being provided by a weight whose fall is controlled by the escapement.



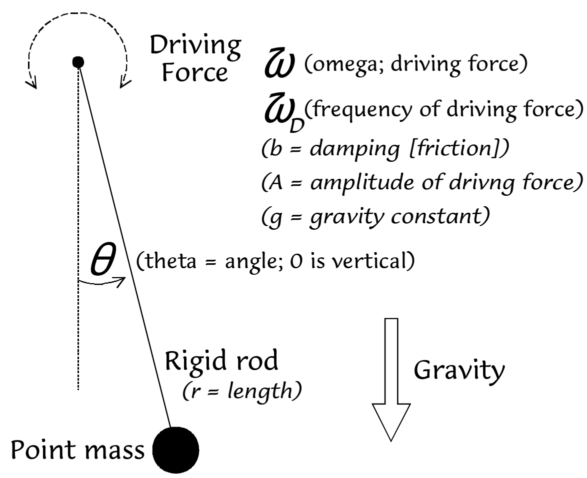
Experimental pendulums swing 360º

Pendulum models are different from clock pendulums. The weight is a point mass (all the weight is considered to reside at one point), the rod is rigid, the pendulum swings in a 360º arc (takes some getting used to), and the kick can come at any time in the swing, and at any frequency. Pendulum behavior is influenced by many factors; the most common ones include:

1. A or ω = drive amplitude, or driving force, or torque
2. ω_D = drive frequency - how often the kick comes, which may not be the same as swing frequency.
3. b = damping constant (friction), meaning a pendulum will slow down over time if there is no driving force.

In addition, these also influence pendulum behavior.

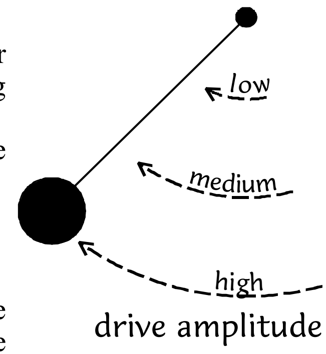
4. R = length of rod
5. g = gravitational constant
6. m = mass of pendulum
7. θ = angle of pendulum (0 = vertical)



Not all combinations of these lead to chaotic behavior; some result in repeating behavior—the ticktock of the grandfather clock, or double loop, or quadruple loop cycles—although even this contains flicker noise. But, if one of the parameters is slowly increased, the behavior of the pendulum can be observed to go through period doublings, or bifurcations, and finally enter chaos, a strange attractor.

For our studies we concentrate on the three parameters: driving force, driving frequency, and damping. Just these three provide many control combinations you can spend hours experimenting with. Different programs or Applets (a computer program that works in a web browser) provide different value ranges for each parameter, and use different symbols for them so be careful if making comparisons from program to program.

Amplitude of Driving Force:



- Low amplitude is close to the pendulum pivot, and high amplitude is closer to the point mass. The closer to the pivot point the less energy the driving force imparts to the pendulum.
- Driving force alternates between clockwise motion and counter clock wise motion.

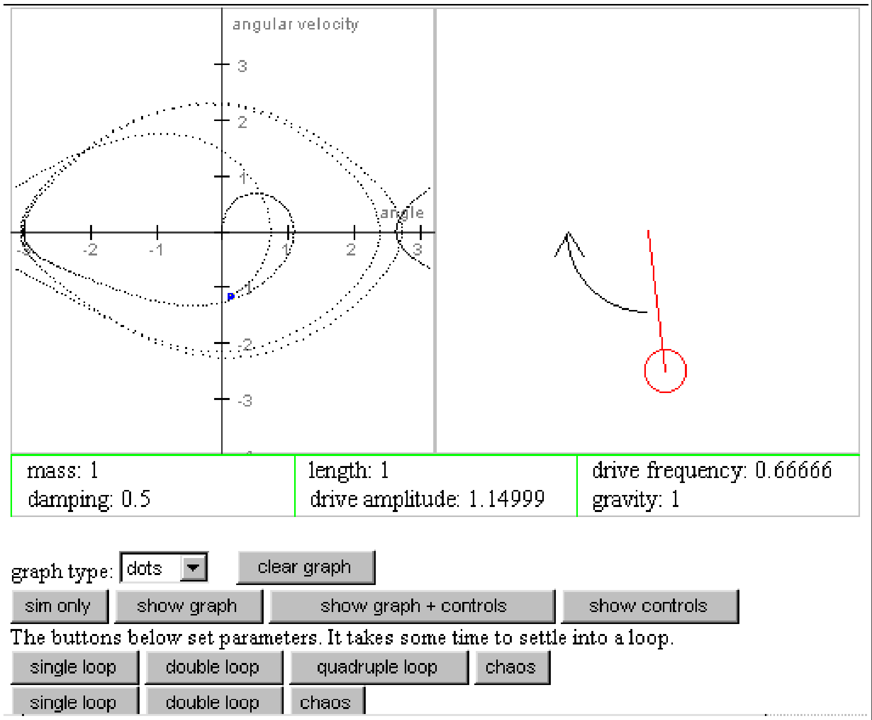
Drive Frequency:
- How fast the driving force switches from clockwise to counter clockwise.
- In some circumstances the drive frequency and the pendulum swing are synchronized, but more commonly the pendulum is swinging one direction while the driving force is moving the opposite direction, slowing the pendulum down rather than kicking it.

Damping:
- Friction. In a real system there are several sources of friction, including air resistence and moving contact between mechanical parts. In computer models the damping factor adds friction. Of course, it is possible to set damping to zero, in which case the system becomes frictionless. If damping is set to zero while the pendulum is in motion it becomes a perpetual motion machine.

❑ Our goals here are two fold:
1. To demonstrate a pendulum's behavior undergoes bifurcations and can be chaotic. In fact, that its behavior has the same range of responses as the logistic equation X_{next}, including point, limit cycle, and strange attractors. The extensibility is that any oscillating system is equally capable of this spectrum of behavior.
2. To develop familiarity with ways of exhibiting a system's behavior: time series, phase spaces, Poincare sections, etc.

Experiment One - The Driven Pendulum
Attractors in Phase Space

- ❑ Go to this web site: http://www.myphysicslab.com/pendulum2.html. There is an Applet (a computer program that works in a web browser) of a driven pendulum.
- ❑ On the Right: a pendulum that swings 360°. Driving force arrow shows direction and power (length of arrow). Note the arrow sometimes points opposite direction pendulum is swinging; this slows down the pendulum.
- ❑ Left: phase space of angular velocity and position (angle in radians).
- ❑ Watch the simulation for a while; can you match movement of the pendulum with the trajectory in phase space? This is very hard to do. But, if you trust that the calculation are being done right, you can just trust and compare the phase diagrams. If you want to correlate the pendulum swing with the phase space diagram, on the next page is an analysis of the single loop condition showing the matching positions on the trajectory with pendulum positions and movements.



- ❑ **Changing parameters**. All parameters can be changed. Click on them and they turn red; type a new value and strike Enter.

1. Run One - Comparing Phase Space Attractors

☞ **Single Loop** - click the top left single loop button and give the program a dozen or so cycles to settle in. Sketch the phase space in the table below.

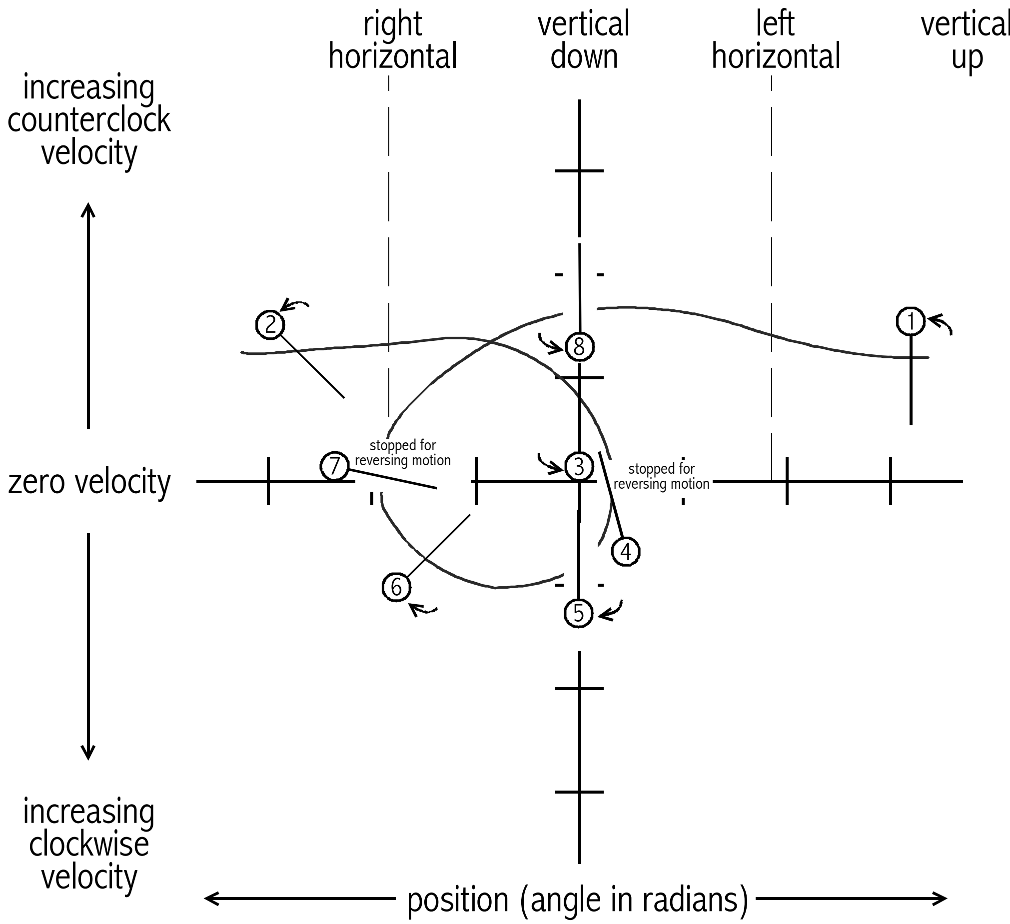
- ‣ Or, if you prefer, screen capture the image (Alt/Print Screen) and print it. If you want to do this but don't know how just ask and we will show you.
- ‣ After it settles, if you click the "clear graph" button you can get rid of the settling in paths.

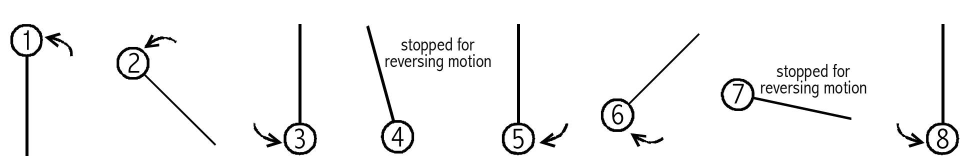
☞ Do the same for the **Double Loop**, **Quadruple Loop**, and **Chaos**. Let the Chaos one run for several dozen cycles before capturing it.

The 360 Degree Pendulum in Phase Space

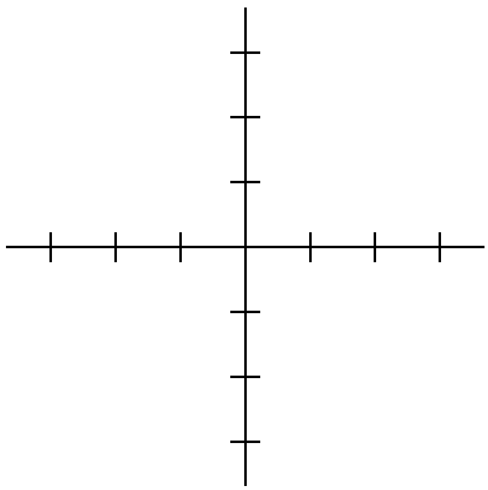
Observing a 360 degree pendulum, and comparing its behavior with the phase space trajectory takes a lot of practice, and can be mind numbing. We suggest that you trust the computer and not try to figure it out since it takes a lot of observation time and practice. But, if you want some insight into how to compare the two the phase space diagram below for a limit cycle attractor shows the pendulum position and direction of motion at various points on the phase space.

Note that the upper half is counterclockwise motion and the lower half clockwise motion. Also note that the trajectory on the center line always means the pendulum is in the down position, while trajectory on the far left/right means the pendulum is in the up position. When the pendulum is horizontal the phase space trajectory is half way between the up and down positions.

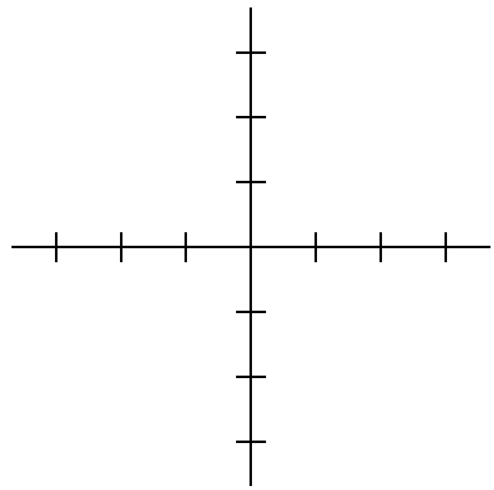




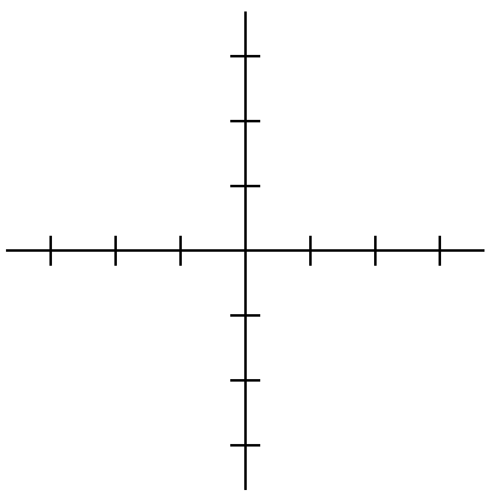
1. Experimental Record One – Comparing Phase Space Attractors



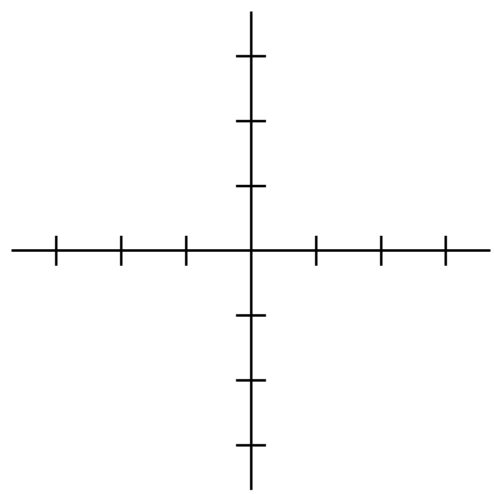
Single loop



Double Loop



Quadruple Loop



Chaos

Which systems settle into a perfect limit cycle attractor? Which do not?

How long does it take for each attractor to settle into its pattern?

Which attractors show sensitive dependence? How do you know?

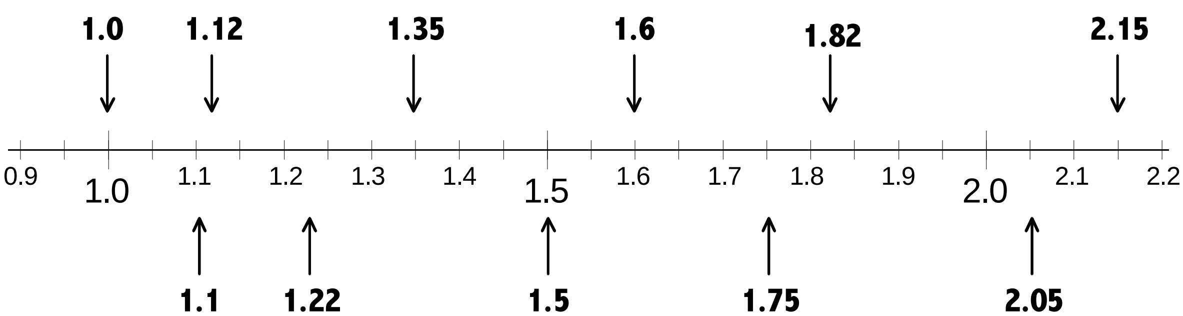
How are each of these attractors related to bifurcations?

Experiment Two - The Driven Pendulum
Bifurcations in Phase Space

Now that we have some idea how to read pendulum behavior in a Poincare section we want to go back and systematically observe changing behavior in phase space. The "MyPhysicsLab" applet allows us to do that easily.

2. Run Two - Bifurcations in Phase Space

- ❑ Return to this web site: **http://www.myphysicslab.com/pendulum2.html**.
  - ☞ Click on the "drive amplitude" box; it will turn red. Type in a new value and strike Enter to set the value.
- ❑ On the number line below are eleven amplitude values. Beginning with 1.0, type the value and wait until the attractor settles down. Then, in the boxes two pages over sketch and/or describe the final settled attractor. Clear the graph if you need to to see just the final state.
  - ☞ If you want you can screen capture the final attractor states to compare them.

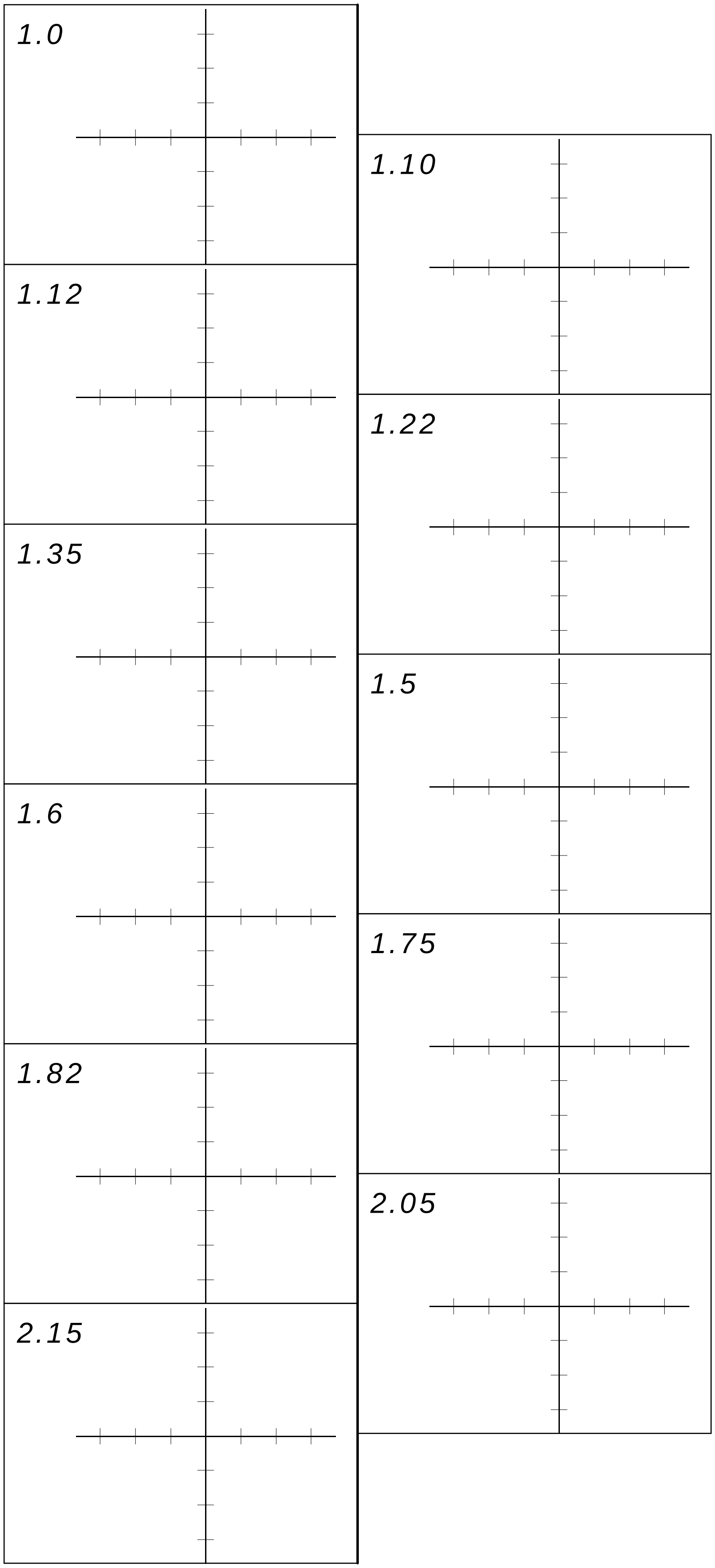


3. Experimental Record Three - Bifurcations in Phase Space

Compare the attractors in the left column (1.0, 1.12, 1.35, etc.) with those in the right column (1.1, 1.22, 1.5, etc.) Describe how they compare.

Explain the relationships you see in the changing attractors.
Now, look at the diagram on the last page. What is the relationship between your observations in this experiment and that diagram?
Describe the relationships between the logistic equation X-next and the behavior of a driven pendulum. How are they similar? How are they different?

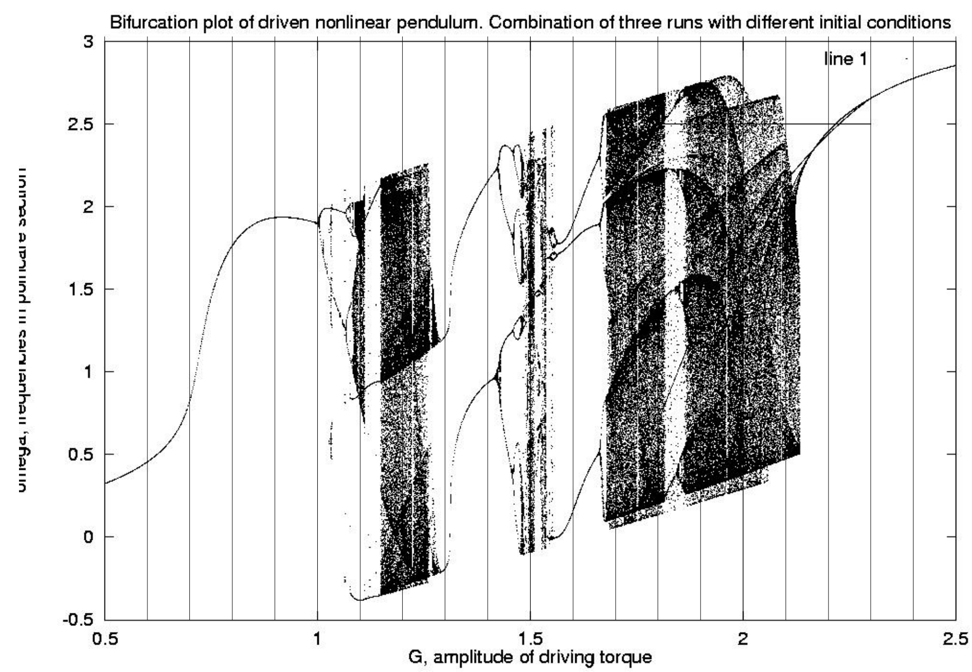
Got It?


1.0
1.10
1.12
1.22
1.35
1.5
1.6
1.75
1.82
2.05
2.15

Bifurcation Diagram From:

THE NONLINEAR PENDULUM: PHSI 362 PROJECT

Jan Max Walter Krüger
University of Otago, New Zealand
http://hubble.physik.uni-konstanz.de/jkrueger/phsi362/index.html



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