# Laboratory Experiments With The X-next Equation <br> Using the Model of British Biologist Rollo May, and M.J. Feiqenbaum Programmed by Stephen J. Baedke 

GenSci 104 - Artificial Life, Chaos, and Complexity
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## Purpose

We have already had a class demonstration on the X-next equation, but we want you to explore some of the subtleties of this equation on your own.

Notes and Comments
() When it comes time to do the formal experiments follow the instructions below. They are designed to systematically lead you through a series of observations.
(-) You may consult with your instructor about questions that come up in your experiments.
© You may discuss your experiments with other class members, but the results in your ARecord of Experimental Results@must follow logically from your own observations and specific experimental results.

## Opening the X nex Program

- X-next is available only in the Geology Department computer lab, Miller 232. All the computers have the program.
- Turn the computer and monitor on. It will open to a Windows 95 screen.
- When the Windows Program Screen comes open:

Click on the A-Life icon to bring that window up front.
Double click the $\mathbf{X}$-next icon to bring up the X-next program.

Both the Instructions for the experiments and the spaces for recording your experimental results are contained here.

## Exploring the X-next Proqram

- X-next is a Windows based program.
$\square$ If the program is unfamiliar to you observe the following:
Values of X from 0.0 to 1.0 are along the vertical ( Y ) axis.
Number of equation iterations are along the horizontal (X) axis. The actual number of iterations is in the lower left box on the screen.
The A@value and Anitial x@alue are also in the lower left.
Overwrite Previous means that the curve for the last run will be erased before the new curve is drawn. Turning this to $\mathrm{ANo} @ v i l l$ allow you to compare curves.

Calculation Interval - default is 5. To make the calculations go more quickly head toward a value of zero.

View Output gives you all the calculated values for the iterations you have.

## Experiment One- X-next Attenuation

## To attenuate is to diminish to extinction (zero) or to some stable value.

We observed during the lecture demonstration that up to a certain value of $A @ h e ~ e q u a t i o n ~ l e a d s ~$ to attenuation; beyond that it might not. For 100 iterations find this value.

## 1. Run One

Open X-next to the default values of $\mathbf{r}=\mathbf{2 . 7}$ and Iterations= $\mathbf{1 0 0}$.
Overwrite Previous = No
Increase values of" $\mathrm{r}^{\prime \prime}$ from 2.7 to $\mathbf{3 . 1}$ by values of $\mathbf{0 . 1}$.

| $r=2.7$ | $r=2.8$ | $r=2.9$ | $r=3.0$ | $r=3.1$ |
| :--- | :--- | :--- | :--- | :--- |

Describe what is happening to the attenuation.

Do you think that@@ 3.1 will ever attenuate? Why or why not?

Imagine that each iteration represents a year of time, and we are observing a natural phenomena, say the likelihood the conditions next year will bring a major flood. Now, 100 years is about the time span we humans have had to observe natural phenomena in order to make predictions.

Based on these observations of 100 iterations, how much would you trust a prediction for values between 3.0 and 3.1. Why?

## 2. Run Two - Isolating Attenuation at $\mathbf{R}=\mathbf{3 . 0}$ <br> Set" $\mathbf{r "}$ at $\mathbf{3 . 0}$ and iterations at $\mathbf{5 0 0}$. <br> Set Calculation Interval to zero (the fastest calculation speed). <br> Overwrite Previous = Yes

Do 500 iterations at " $\mathbf{r} "=\mathbf{3 . 0}$ demonstrate that attenuation will happen?

Go to VIEW OUTPUT. You will see the values of A@or every iteration from 1-500. Go to iterations 480-500. Has the number stabilized (to 2 decimal points), or is it still oscillating?

Do 1000 iterations demonstrate that attenuation will happen? At what iteration?

## 3. Run Three - Isolating Attenuation at $\mathbf{R}=\mathbf{3 . 1}$

Set"r" at $\mathbf{3 . 1}$ and iterations at $\mathbf{1 0 0 0}$.
Set Calculation Interval to zero (the fastest calculation speed).
Overwrite Previous = Yes

Do 1000 iterations demonstrate that $\mathrm{A} @ 3.1$ will attenuate?

Do you think it is necessary to run more than 1000 iterations to check for attenuation? Is so, set new iteration values and report what happens.
_ Iterations
$\qquad$ Iterations
$\qquad$ Iterations

## 4. Run Four - Isolating Attenuation Between Values

Clearly, A@2.99 leads to attenuation while 3.1 leads to continuing oscillation (first bifurcation to two values of 0.55 and 0.76 ; notice this is true out to all 15 decimal places.)

We want you to try to find the exact value of A.@between 2.9999 and 3.1000 where it stops attenuating and period two oscillation begins. Begin with values of A@o two decimal places (e.g. 3.02), but increase the decimal places as you need. The two values of $X$ should repeat out to 4 decimal places.

Set Iterations at 500.
Set CALCULATION InTERVAL to zero (the fastest calculation speed).
Overwrite Previous = Yes.

Experiment with values between 3.0 and 3.1 ( 500 iterations only) trying to isolate where period 2 bifurcation initiates. Use as many decimal places for A@as you need to do this; get x values to 4 decimal places. The spaces below are just to help you keep track of your explorations. If you change the number of iterations, record that also.

1. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
2. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
3. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$
4. $\mathrm{r}=$ $\qquad$ $X=$ , $\qquad$
5. $\mathrm{r}=\ldots \mathrm{X}=$ $\qquad$ , $\qquad$
6. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
7. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
8. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
9. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$
10. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
11. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
12. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ ,
13. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ ,
14. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$
15. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$
16. $\mathrm{r}=\quad \mathrm{X}=$ $\qquad$ ,

At what iteration did attenuation occur? And what value of $x$ ?

To how many decimal places of $X$ were you checking this? Do you think it makes a difference in your analysis? What would be required to get this value to, say, 4 decimal places?

Sensitive Dependence on Initial Conditions: In terms of this experiment describe what sensitive dependence means.,

## Experiment Two-X-next Period Four Bifurcation

## 5. Run Five - Finding the Period Four Bifurcation

OK, to the best of our ability, in a reasonable amount of time, we have isolated the period 2 bifurcation. Now we want you to try to find the Period 4 bifurcation.

Design this experiment any way you want to isolate this A@alue. We have provided below 30 spaces to keep track of your experiment. Note that as you go to higher and higher bifurcation values it usually takes more and more decimal places of A.@

1. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$
$\qquad$ ; Iteration $\qquad$
2. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
3. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
4. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
5. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
6. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
7. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
8. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
9. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , $\qquad$ , $\qquad$ ; Iteration $\qquad$
10. $\mathrm{r}=$ $\qquad$ $X=$ $\qquad$ , , , _ ; ; Iteration $\qquad$


## Experiment Three- X-next Period Eight Bifurcation

## 6. Run Six - Period Eight Bifurcation

OK, OK - much more challenging. Try to find the period 8 bifurcation.
Design this experiment any way you want to isolate this A@alue. We have provided below 30 spaces to keep track of your experiment.


21 $\qquad$ $\mathrm{X}=$ $\qquad$ , , __, $\qquad$ , $\qquad$ , $\qquad$ _
22. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , _ , $\qquad$ ——, $\qquad$ $-$ $\qquad$
$\qquad$
$\qquad$
23. $\mathrm{r}=\ldots \mathrm{X}=$ $\qquad$ , _, $\qquad$
$\qquad$ , __, $\qquad$
$\qquad$
$\qquad$
24. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , $\qquad$ , $\qquad$ , __, , $\qquad$
$\qquad$
25. $\mathrm{r}=$ $\qquad$
$\qquad$ , —, $\qquad$
$\qquad$ , —, $\qquad$
26. $\mathrm{r}=$ $\qquad$
$\qquad$ , $\quad$,
$\qquad$ , ——, -
27. $\mathrm{r}=$ $\qquad$
$\qquad$ , —,
$\qquad$ , $\qquad$ -
$\qquad$
28. $\mathrm{r}=$ $\qquad$ $\mathrm{X}=$ $\qquad$ , , $\qquad$ , _ , $\qquad$ , ,
29. $\mathrm{r}=$ _ $\mathrm{X}=$ $\qquad$ , _, $\qquad$
$\qquad$ , _ , $\qquad$
$\qquad$ -


Now, print out the last 40 values of $\mathbf{A X}$ @enough to fit on one sheet of paper). You can do this with the following steps.
(1) Go to View Output in X-next.
(2) Select the last 40 values of $\mathbf{A X} @ h o l d$ down the left mouse button and drag across to highlight what you want) and do an EDIT: COPY.
(3) Open a new Notepad (Start: Programs: Accessories: Notepad) and paste these values into it.
(4) Go To: File: Print

On the printout, starting at the bottom, find and bracket 8 values of $\mathcal{A X} @$ From there count off 8 more and bracket them. Continue through all 40 values of $\mathrm{AX} @$
Question?: How precisely do the eight numbers repeat?

Now go back and carefully compare the bundles of 8 numbers. Do all bundles, in fact, repeat more or less the same numbers?

Alternatively, can you find any sets of $2,3,4$, or 5 numbers which seem to break the rhythm of eights. Is so, isolate them and go back and carefully bracket only those bundles of 8 that repeat, separately bracketing the smaller bundles.

All of you may not have found smaller bundles of numbers. If you did not, go back and slowly increase the value of A-@until they appear.

Can you explain why these smaller bundles of numbers appear? What is happening?

If the natural world behaves the way the $x$-next equation behaves, what does it say about our ability to predict the future behavior of natural phenomena?

## Experiment Four - X-next Extinction, or Breaking the Bank

## 7. Run Seven - Extinction

This last experiment asks you to explore the upper reaches of A@ How precisely can you find the value of A@hat causes the x-next program to lock up?


Can you provide any explanation for what is happening when X-next locks up?
$\square$
$\odot \mathbf{F} \odot \mathbf{i} \odot \mathbf{n} \odot \mathbf{i} \odot \mathbf{S} \odot$

